SOLVED PAPER

OR OPERATION RESEARCH

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DECENT PHOTOCOPY

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(خریدے ہوئے نوٹس واپس یا تبدیل نہیں ہوں گے) شکریہ

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A/2016.

Q A company manufactures two types of product, A and B and sells them at a profit of Rs. 4 on type A and Rs. 5 on type B. Each product is processed on two machines X and Y. Type A requires 2 minutes of processing time on X and 3 minutes of processing time on Y. And type B requires 2 minutes of processing time on X and 2 minutes of processing time on Y. The machine X, is available for not more than 5 hours and 30 minutes while machine Y is available for 8 hours.

a) Formulate the problem as Linear programming model.

b) Solve the problem using the Graphical Method.

Sol. a)

Step I. Decision Variable.

let x denotes the total number of ~~chairs~~ products of type A and y denotes the total number of products of type B.

Step II. Objective function.

Profit: $z = 4x + 5y$.

Step III. Constraints.

	X	Y	Profit
A	2	3	4
B	2	2	5
Availability of Machines	5h 30min = 330 min	8hrs. = 480 min	—

$2x + 2y \leq 330$

$3x + 2y \leq 480.$

$x, y \geq 0.$

(b)

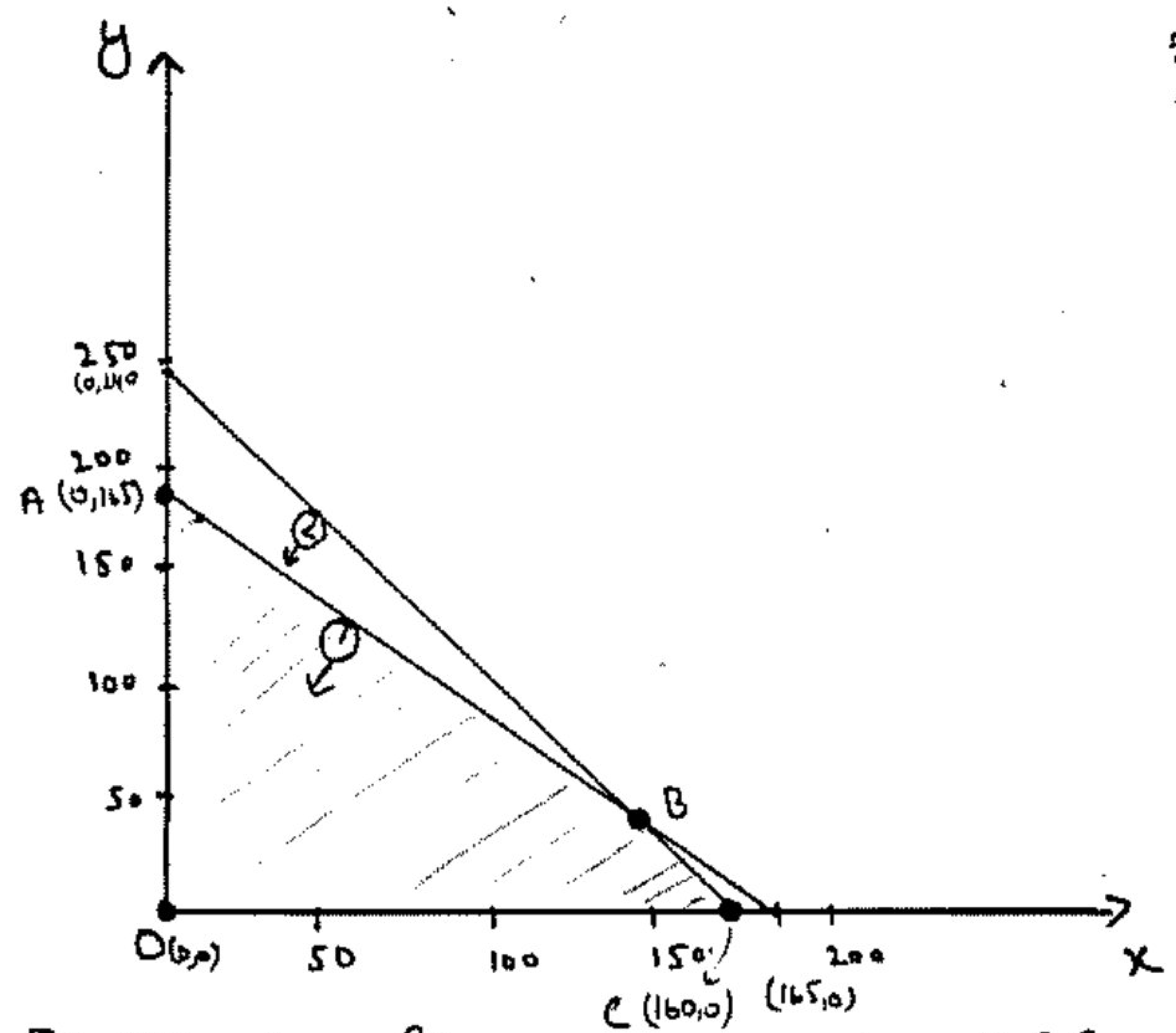
Solution of LP model by Graphical Method

$Z = 4x + 5y.$

$x + y \leq 165$ (1)

$3x + 2y \leq 480.$ (2)

$x, y \geq 0.$



Feasible reigon of the given problem is OABC.

Corner points.

$O = (0,0)$, $A(0,165)$, $B = ?$, $C = (160,0)$

For point B.

$x + y = 165$ (1)

$3x + 2y = 480$ (2)

$3 \times$ (1) $-$ (2)

$3x + 3y = 495$

$\pm 3x \pm 2y = \pm 480$

$y = 15$ put in (1)

$x + 15 = 165$

$x = 150$

Thus point $B = (150, 15)$.

③

For point B.

$$2x + 5y = 800 \quad (1)$$
$$2x + y = 400 \quad (2)$$

(1) − (2)

$$4y = 400$$
$$y = 100 \text{ put in } (2)$$

$$2x + 100 = 400$$
$$2x = 300$$
$$x = 150$$

Thus B(150, 100).

Corner points	$z = x+y$.
O(0,0)	$z=0$
A(0,160)	$z=160$
B(150,100)	$z=250$
C(200,0)	$z=200$.

Thus maximum number of fish that the lake can support is 250.

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Q Use graphical method to solve the problem.

Max. $z = 5x_1 + 7x_2$

Subject to $x_1 + x_2 \le 4$

$3x_1 + 8x_2 \le 24$

$10x_1 + 7x_2 \le 35$

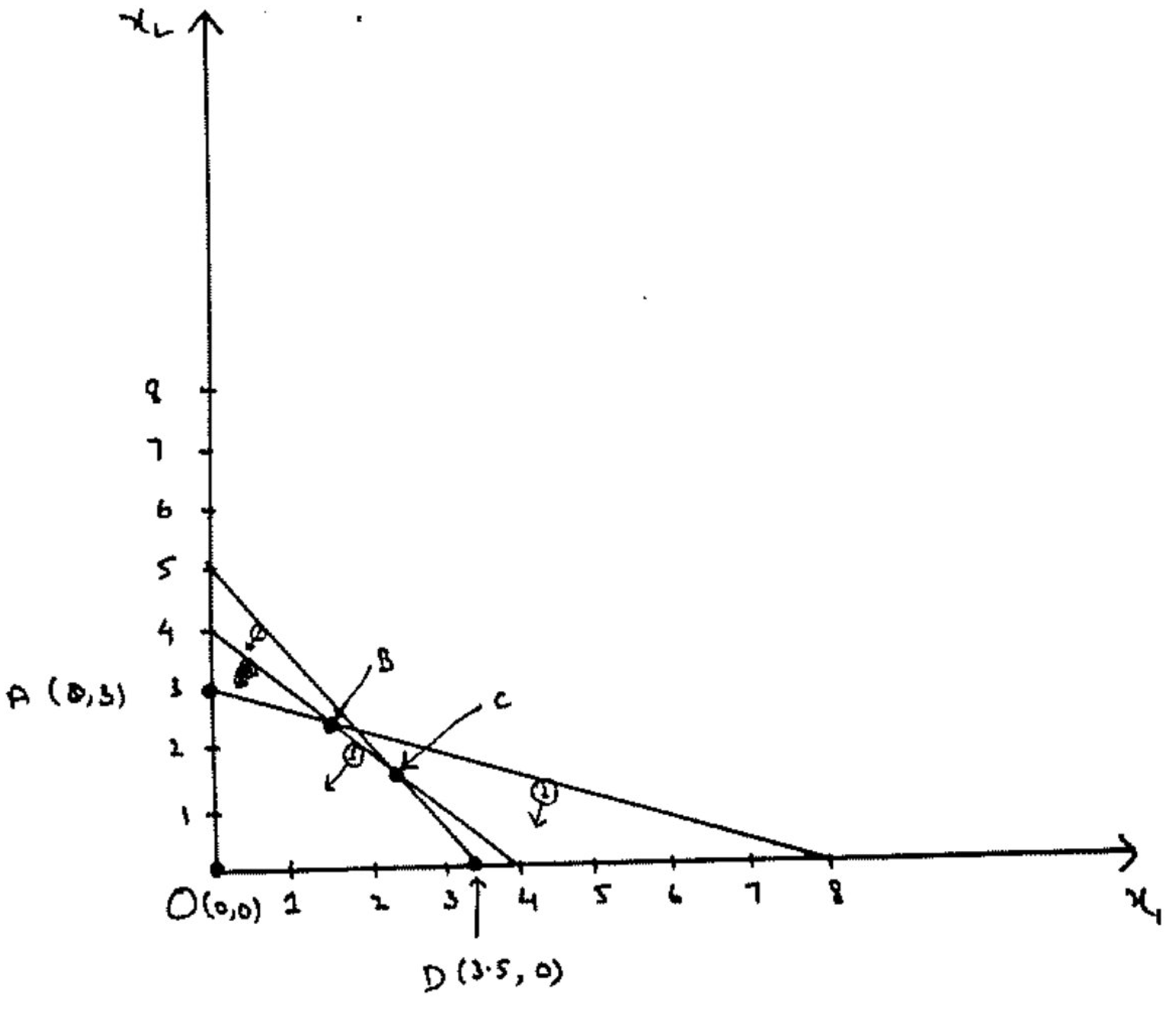
$x_1, x_2 \ge 0$.

Sol.

The given LP model is

Max : $z = 5x_1 + 7x_2$

Subject to
$x_1 + x_2 \le 4$ (1)
$3x_1 + 8x_2 \le 24$ (2)
$10x_1 + 7x_2 \le 35$ (3)
$x_1, x_2 \ge 0$



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Corner points	$z = 4x + 5y$
O (0,0)	$z = 0$
A (0,165)	$z = 825$
B (150,15)	$z = 675$
C (160,0)	$z = 640$.

The profit function $z = 4x + 5y$ is maximum at the corner point A(0, 165).

A/2015.

Q. A lake contains two type of fishes, I and II. The owner provides two types of foods, A and B for these fish. Species I requires 2 units of food A and 4 units of food B; and species II requires 5 units of food A and 2 units of food B. If the owner has 800 units of each food, find the maximum number of fishes that the lake can support.

a) Formulate the problem as LP model.

b) Solve the problem using the Graphical method

Sol. (a)

Step I Decision variables.

let x denotes the fish of species I and y denotes the fish of species II.

Step II: Objective function.

Max : $z = x + y$.

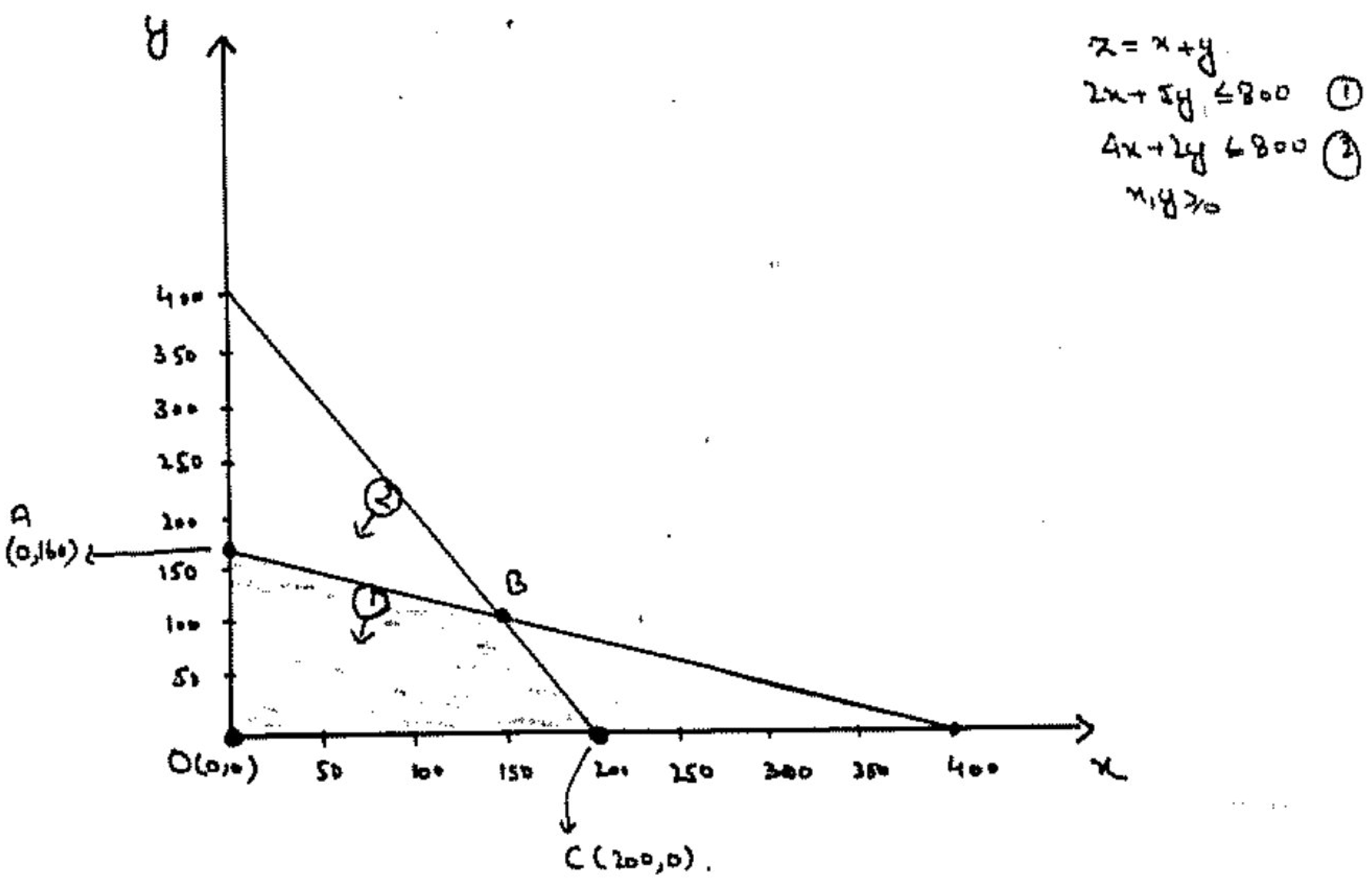
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Step III. Constraints.

	Food type	
	A	B
Fish I (x)	2	4
Fish II (y)	5	2
Availability of food	800	800

$$2x + 5y \le 800 \quad (1)$$
$$4x + 2y \le 800 \quad (2)$$
$$x, y \ge 0$$

(b) Solution of LP model by Graphical Method.



The feasible reigon is OABC.

Corner points.

O (0,0), A(0,160), B = ?, C(200,0)

A/2016, A/2013.

Q Apply Simplex Method to solve the following problem

Minimize : $z = x_1 - 3x_2 + 2x_3$

Subject to:

$$3x_1 - x_2 + 3x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10.$$
$$x_1, x_2, x_3 \geq 0.$$

Sol

Writing the LP model is standard form by introducing slack variables

Max $z^* = (-z) = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to:

$$3x_1 - x_2 + 3x_3 + s_1 = 7$$
$$-2x_1 + 4x_2 + s_2 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10.$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

Writing the LP model in tableau form

Basic	x_1	↓ x_2	x_3	s_1	s_2	s_3	sol	ratio
Z^*	1	-3	2	0	0	0	0	—
s_1	3	-1	3	1	0	0	7	$7/-1 = -7$
← s_2	-2	[4]	0	0	1	0	12	$12/4 = 3$
s_3	-4	3	8	0	0	1	10	$10/3 =$

Optimality condition implies that x_2 is entering variable

∞

Basic	↓ x_1	x_2	x_3	s_1	s_2	s_3	sol	ratio	
Z^*	$-1/2$	0	2	0	$3/4$	0	9		$R_1 + 3R_3$
← s_1	[$5/2$]	0	3	1	$1/4$	0	10	$10 \times \frac{2}{5} = 4$	$R_2 + R_3$
x_2	$-1/2$	1	0	0	$1/4$	0	3		
s_3	$-5/2$	0	8	0	$-3/4$	1	1		$R_4 - 3R_3$

Optimality condition implies that x_1 is entering variable

Basic

Z^*

x_1

x_2

s_3

$\frac{1}{10} \times \frac{5}{2}$

Basis	x_1	x_2	x_3	S_1	S_2	S_3	sol.	
Z^*	0	0	13/5	1/5	4/5	0	11	$R_1 + \frac{1}{2} R_2$
x_1	1	0	6/5	2/5	1/10	0	4	
x_2	0	1	3/5	1/5	3/10	0	5	$R_3 + \frac{1}{2} R_2$
S_3	0	0	11	1	-1/2	1	11	$R_4 + \frac{5}{2} R_2$

Since there is no coefficient in the objective row with most -ve, so the solution is optimum.

$Z = 11$, $x_1 = 4$, $x_2 = 5$, $x_3 = 0$

A/2014

Q. Consider the following linear program

Maximize $z = 16x_1 + 15x_2$

Subject to

$$40x_1 + 31x_2 \leq 124$$

$$-x_1 + x_2 \leq 1$$

$$x_1 \leq 3$$

$$x_1, x_2 \geq 0.$$

Solve the problem by Simplex Method.

Sol. Writing the LP model in standard form by introducing slack variables

Max: $z = 16x_1 + 15x_2 + 0S_1 + 0S_2 + 0S_3$

subject to

$$40x_1 + 31x_2 + S_1 = 124$$

$$-x_1 + x_2 + S_2 = 1$$

$$x_1 + S_3 = 3$$

$$x_1, x_2, S_1, S_2, S_3 \geq 0.$$

Writing the LP model in tableau form

Basic	x_1 ↓	x_2	S_1	S_2	S_3	Sol	ratio
z	-16	-15	0	0	0	0	–
S_1	40	31	1	0	0	124	$\frac{124}{40} = 3.1$
S_2	-1	1	0	1	0	1	–
← S_3	[1]	0	0	0	1	3	$3/1 = 3$

Optimality condition implies that x_1 is entering variable

Basic	x_1	x_2 ↓	S_1	S_2	S_3	Sol	ratio	
z	0	-15	0	0	16	48	–	$R_1 + 16R_4$
← S_1	0	[31]	1	0	-40	4	$4/31$	$R_2 - 40R_4$
S_2	0	1	0	1	1	4	$4/1$	$R_3 + R_4$
x_1	1	0	0	0	1	3	–	

Optimality condition implies x_2 is entering variable

Basic	x_1	x_2	S_1	S_2	S_3	Sol		Enter Basic
z	0	0	15/31	0	-104/31	1548/31	$R_1 + 15R_2$	z
x_2	0	~~31/40~~ 1	1/31	0	-40/31	4/31		S_1
S_2	0	0	-1/31	1	71/31	120/31	$R_3 - R_2$	← x_2
S_1	1	0	0	0	1	3	R_4	S_3

Since no coefficient in the z-row is negative so the solution is optimum

$\therefore$ $z = \frac{1548}{31}$, $x_1 = 3$, $x_2 = \frac{4}{31}$.

Now we solve the problem by taking least -ve in the 1st tableau.

Basic	x_1	x_2 ↓	S_1	S_2	S_3	Sol	ratio
z	-16	-15	0	0	0	0	—
S_1	40	31	1	0	0	124	124/31
← S_2	1	[1]	0	1	0	1	1
S_3	1	0	0	0	1	3	—

Basic
z
S_1
x_1
S_3

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Entering variable is x_1 and leaving variable is s_2.

Basic	x_1 ↓	x_2	s_1	s_2	s_3	sol	ratio	
z	-1	0	0	15	0	15	—	R_1+15R_3
s_1	9	0	1	-31	0	93	93/9	R_2-31R_3
← x_2	[1]	1	0	1	0	1	1/1	
s_3	1	0	0	0	1	3	3/1	

Feasibility condition implies x_2 is leaving variable.

Basic	x_1	x_2	s_1	s_2	s_3	sol
z	0	0	0	16	0	16
s_1	0	-9	1	-40	0	84
x_1	1	1	0	1	0	1
s_3	0	-1	0	-1	1	2

Since there is no least -ve

So $z=16$, $x_1=1$, $x_2=0$.

Conclusion

The number of iterations in both cases are same.

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A/2011.

Q. Use simplex method to solve the following problem.

Max. $Z = 3x_1 + 2x_2 + 5x_3$.

Subject to: $x_1 + x_2 + x_3 \leq 9$

$2x_1 + 3x_2 + 5x_3 \leq 30$

$2x_1 - x_2 - x_3 \leq 8$.

$x_1, x_2, x_3 \geq 0$.

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Sol. Writing the LP model in standard form by introducing slack variables

Max : $Z = 3x_1 + 2x_2 + 5x_3 + 0S_1 + 0S_2 + 0S_3$

subject to $x_1 + x_2 + x_3 + S_1 = 9$.

$2x_1 + 3x_2 + 5x_3 + S_2 = 30$

$2x_1 - x_2 - x_3 + S_3 = 8$

$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$.

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Writing the LP standard model in tableau form

Basic	x_1	x_2	x_3 ↓	s_1	s_2	s_3	sol	ratio
z	-3	-2	-5	0	0	0	0	—
← s_1	1	1	[1]	1	0	0	9	$9/1 = 9$
s_2	2	3	5	0	1	0	30	$30/5 = 6$
s_3	2	-1	-1	0	0	1	8	—

Optimality condition implies that x_3 is entering variable.

Basic	x_1	x_2	x_3	s_1	s_2	s_3	sol	
z	2	3	0	5	0	0	45	$R_1 + 5R_2$
x_3	1	1	1	1	0	0	9	
x_2	-3	-2	0	-5	1	0	-15	$R_3 - 5R_2$
s_3	3	0	0	1	0	1	17	

Since no co-efficient of non-basic variable is -ve so the solution is optimal

$z = 45, \quad x_3 = 9, \; x_2 = 0, \; x_1 = 0$

A/2011

Q Use Simplex method to solve

Minimize $z = 2x_1 + 3x_2$

subject to $x_1 - 2x_2 \le 0$

$-2x_1 + 3x_2 \ge 6.$

x_1, x_2 unrestricted.

Sol. Write the system in standard form

Max. $z^* = (-z) = -2x_1 - 3x_2$

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Dual Simplex Method.

A/2017

Q. Solve by using Dual Simplex Method.

Max: $z = 5x_1 + 6x_2$

Subject to. $x_1 + x_2 \geq 2$

$4x_1 + 2x_2 \geq 4$

$x_1, x_2 \geq 0.$

Sol. Step I

Write the constraints in '≤' form

$-x_1 - x_2 \leq -2$

$-4x_2 - 2x_2 \leq -4$

$x_1, x_2 \geq 0.$

Write the LP model in standard form.

Max: $z = 5x_1 + 6x_2 + 0s_1 + 0s_2$

$-x_1 - x_2 + s_1 = -2$

$-4x_2 - 2x_2 + s_2 = -4.$

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Step II Put the problem in tableau form

Basic	x_1 ↓	x_2	s_1	s_2	sol
z	-5	-6	0	0	0
s_1	-1	-1	1	0	-2
← s_2	$\boxed{-4}$	-2	0	1	-4
	$\left\|\frac{-5}{-4}\right\| = \frac{5}{4}$	$\left\|\frac{-6}{-2}\right\| = 3$	$-$	$-$	

∞

Feasibility condition ⇒ s_2 is leaving

Optimality condition ⇒ x_1 is entering

Basic	x_1	x_2	s_1	s_2 ↓	sol	
z	0	$-7/2$	0	$-5/4$	5	$R_1 + 5R_3$
← s_1	0	$-1/2$	1	$\boxed{-1/4}$	-1	$R_2 + R_3$
x_1	1	$1/2$	0	$-1/4$	1	
Ratio		$\left\|\frac{-7}{2} \times (-2)\right\| = 7$		$\left\|\frac{-5}{4} \times (-4)\right\| = 5$		

S_2 leaves and s_2 enters the system

Basic	x_1	x_2	s_1	s_2	sol	
z	0	-1	-4	0	10	$R_1 + \frac{1}{4} R_2$
s_2	0	2	-4	1	4	
x_1	1	1	-1	0	2	$R_3 + \frac{1}{4} R_2$

Since no co-efficient in the solution column is -ve. Hence the solution is optimum.

$z = 10, \quad x_1 = 2, \quad x_2 = 0.$

A/2016.

Q Apply Dual Simplex Algorithm to solve

Minimize: $z = x_1 + x_2$.

Subject to: $2x_1 + x_2 \geq 2$

$-x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0.$

Writing the constraints in standard form

$$-2x_1 - x_2 \leq -2$$

$$x_1 + x_2 \leq -1$$

$$x_1, x_2 \geq 0$$

Write the LP model in standard form by introducing slack variables

Min: $z = x_1 + x_2$

subject to $-2x_1 - x_2 + S_1 = -2$

$x_1 + x_2 + S_1 = -1$

$x_1, x_2, S_1, S_2 \geq 0$

Writing in tableau form.

Basic	↓ x_1	x_2	S_1	S_2	sol
z	-1	-1	0	0	0
← S_1	[-2]	-1	1	0	-2
S_2	1	1	0	1	-1
ratio	$\left\|\frac{-1}{-2}\right\| = \frac{1}{2}$	$\left\|\frac{-1}{-1}\right\| = 1$	–	–	

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Feasibility condition $\Rightarrow$ S_1 is leaving variable

Optimality condition $\Rightarrow$ x_1 is entering variable.

Basic	x_1	x_2	s_1	s_2	sol.	
z	0	$-1/2$	$-1/2$	0	1	R_1+R_2
x_1	1	$1/2$	$-1/2$	0	1	
$\leftarrow S_2$	0	$1/2$	$1/2$	1	-2	R_3-R_2
ratio	-	-	-	-		

Feasibility condition implies that S_2 is leaving but no coefficient in the S_2-row is -ve.

Hence the solution is optimal

$z = 1, \quad x_1 = 1, \quad x_2 = 0.$

A/2015

Q. Apply Dual Simplex Method.

Minimize: $z = 5x_1 + 6x_2$

Subject to: $x_1 + x_2 \geq 2$

$4x_1 + x_2 \geq 4$

$x_1, x_2 \geq 0.$

Sol. Writing the constraints in standard form (i.e., '≤' form)

$-x_1 - x_2 \leq -2$

$-4x_1 - x_2 \leq -4$

$x_1, x_2 \geq 0.$

Write the LP model in standard form by introducing slack variables

Min : $z = 5x_1 + 6x_2$

Subject to: $-x_1 - x_2 + S_1 = -2$

$-4x_1 - x_2 + S_2 = -4$

$x_1, x_2, S_1, S_2 \geq 0.$

Write in tableau form

Basic	↓ x_1	x_2	s_1	s_2	sol
z	-5	-6	0	0	0
s_1	-1	-1	1	0	-2
← s_2	$\boxed{-4}$	-1	0	1	-4
ratio	$\left\lvert\frac{-5}{-4}\right\rvert = \frac{5}{4}$	$\left\lvert\frac{-6}{-1}\right\rvert = 6$	–	–	–

Feasibility condition implies that s_2 is leaving variable

Optimality condition implies that x_1 is entering variable.

Basic	x_1	↓ x_2	s_1	s_2	sol	
z	0	$-19/4$	0	$5/4$	5	$R_1 + 5R_3$
← s_1	0	$\boxed{-3/4}$	1	$1/4$	-1	$R_2 + R_3$
x_1	1	$1/4$	0	$1/4$	1	
ratio	–	$\left\lvert\frac{-19}{4} \times \frac{-4}{3}\right\rvert = \frac{19}{3}$	–	–	–	

Feasibility condition implies s_1 is leaving variable

Optimality " " x_2 is entering variable.

$\frac{1}{6} - \frac{4}{3}$

Basic	x_1	x_2	s_1	s_2	Sol	
z	0	0	$-19/3$	$-1/3$	$34/3$	$R_1 + \frac{19}{4} R_2$
x_2	0	1	$-4/3$	$-1/3$	$4/3$	
x_1	1	0	$1/3$	$1/3$	$2/3$	$R_3 - \frac{1}{4} R_2$

Since no coefficient in the solution column is negative

So the solution is optimum.

$z = \frac{34}{3}$, $x_1 = \frac{2}{3}$, $x_2 = \frac{4}{3}$

A12014

Q. Solve by Dual Simplex Method.

Minimize: $z = 3x_1 + 2x_2 + x_3$

Subject to: $3x_1 + x_2 + x_3 \geq 3$

$-3x_1 + 3x_2 + x_3 \geq 6$

$x_1 + x_2 + x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$.

Write the constraints in '≤' form

$$-3x_1 - x_2 - x_3 \leq -3$$

$$3x_1 - 3x_2 - x_3 \leq -6$$

$$x_1 + x_2 + x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

Write the LP model in standard form by introducing slack variables.

Minimize: $Z = 3x_1 + 2x_2 + x_3$

Subject to:

$$-3x_1 - x_2 - x_3 + S_1 = -3$$

$$3x_1 - 3x_2 - x_3 + S_1 = -6$$

$$x_1 + x_2 + x_3 + S_3 = 10.$$

$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$$

Writing the LP model in tableau form.

$-1 + \frac{1}{3}$

Basic	x_1	x_2 ↓	x_3	s_1	s_2	s_3	sol
Z	−3	−2	−1	0	0	0	0
s_1	−3	−1	−1	1	0	0	−3
← s_2	3	[−3]	−1	0	1	0	−6
s_3	1	1	1	0	0	1	10
ratio	−	$\left\lvert\frac{-2}{-3}\right\rvert = \frac{2}{3}$	$\left\lvert\frac{-1}{-1}\right\rvert = 1$	−	−	−	

Feasibility condition implies that s_2 is leaving variable.
Optimality condition implies that x_2 is entering variable.

Basic	x_1	x_2	x_3 ↓	s_1	s_2	s_3	sol	
Z	−5	0	$-1/3$	0	$-2/3$	0	4	$R_1 + 2R_3$
← s_1	−4	0	[$-2/3$]	1	$-1/3$	0	−1	$R_2 + R_3$
x_2	−1	1	$1/3$	0	$-1/3$	0	2	
s_3	2	0	$2/3$	0	$1/3$	1	16	$R_4 - R_3$
ratio	$\left\lvert\frac{-5}{-4}\right\rvert = \frac{5}{4}$	−	$\left\lvert\frac{-1}{3} \times \frac{-3}{2}\right\rvert = \frac{1}{2}$	−	$\left\lvert\frac{-2}{3} \cdot -3\right\rvert = 2$	−	−	

Feasibility ⇒ S_1 is leaving variable

Optimality ⇒ x_3 is entering variable

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Sol	
Z	-3	0	0	-1/2	-1/2	0	9/2	$R_1 + \frac{1}{3}R_2$
x_3	6	0	1	-3/2	1/2	0	3/2	
x_2	-3	1	0	1/2	-1/6	0	3/2	$R_1 - \frac{1}{3}R_2$
S_3	-2	0	0	1	0	1	15	$R_4 - \frac{2}{3}R_2$

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Since no co-efficient in the solution column is -ve, Hence the solution table is optimum

$$Z = \frac{9}{2}, \quad x_1 = 0, \quad x_2 = \frac{3}{2}, \quad x_3 = \frac{3}{2}.$$

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A/2012.

Q Solve the problem by Dual Simplex Method.

Minimize $z = y_1 + 0.4y_2$

Subject to. $0.1y_1 + 0.2y_2 \geq 2$

$0.2y_1 + 0.1y_2 \geq 1.7$

$0.1y_2 \geq 0.6$

$0.1y_1 \geq 0.4$

$y_1, y_2 \geq 0$.

Write the constraints in "≤" form

$-y_1 - 2y_2 \leq -20$.

$-2y_1 - y_2 \leq -17$

$-y_2 \leq -6$

$-y_1 \leq -4$

$y_1, y_2 \geq 0$

Writing the LP model in standard form by introducing slack variables

~~Date~~

Min. $z = y_1 + \frac{2}{5} y_2$

Subject to

$-y_1 - 2y_2 + S_1 = -20$

$-2y_1 - y_2 + S_2 = -17$

$-y_2 + S_3 = -6$

$-y_1 + s_4 = -4$

$y_1, y_2, y_3, s_2, s_3, s_4 \geq 0.$

Tableau form

Basic	y_1	y_2	s_1	s_2	s_3	s_4	sol
z	-1	-2/5	0	0	0	0	0
← s_1	-1	[-2]	1	0	0	0	-20
s_2	-2	-1	0	1	0	0	-17
s_3	0	-1	0	0	1	0	-6
s_4	-1	0	0	0	0	1	-4
ratio	$\left\lvert\frac{-1}{-1}\right\rvert = 1$	$\left\lvert\frac{-2 \times -2}{5}\right\rvert = \frac{4}{5}$	–	–	–	–	–

Fea

Opti

Basic	y_1	y_2	s_1 ↓	s_2	s_3	s_4	Sol	
z	$-4/5$	0	$-1/5$	0	0	0	4	$R_1 + 2/5\, R_2$
y_2	$1/2$	1	$-1/2$	0	0	0	10	R_2
← s_2	$-3/2$	0	$\boxed{-1/2}$	1	0	0	-7	$R_3 + R_2$
s_3	$1/2$	0	$-1/2$	0	1	0	4	$R_4 + R_2$
s_4	-1	0	0	0	0	1	-4	
ratio	$\left\lvert \frac{-4}{5} \times \frac{-2}{3} \right\rvert = \frac{8}{15}$		$\left\lvert \frac{-1}{5} \times -2 \right\rvert = \frac{2}{5}$					

So

Feasibility condition ⇒ s_2 leaves

Optimality condition ⇒ s_1 enters

Basic	y_1	y_2	s_1	s_2	s_3	s_4	Sol	
z	$-1/5$	0	0	$-2/5$	0	0	$34/5$	$R_1 + 4/5\, R_3$
y_2	2	1	0	-1	0	0	17	$R_2 + \frac{1}{2} R_3$
s_1	3	0	1	-2	0	0	14	
s_3	2	0	0	-1	1	0	11	$R_4 + 1/2\, R_3$
← s_4	$\boxed{-1}$	0	0	0	0	1	-4	
ratio	$\lvert -\frac{1}{5} \times -1 \rvert = 1$	–	–	–	–	–	–	

Basi

z

y_2

s_1

s_3

y_1

Som

solt

Q

Feasibility ⇒ s_4 leaves
Optimality ⇒ y_1 enters.

$4 + \frac{2}{5}$ 9
$2 + \frac{12}{5}$

Basic	y_1	y_2	s_1	s_2	s_3	s_4	Sol	
z	0	0	0	-2/5	0	0	38/5	$R_1 + \frac{1}{5}R_5$
y_2	0	1	0	-1	0	2	9	$R_2 - 2R_5$
s_1	0	0	1	-2	0	3	2	$R_3 - 3R_5$
s_3	0	0	0	-1	1	2	3	$R_4 - 2R_5$
y_1	1	0	0	0	0	-1	4	

Since no. co-efficient in the solution column is -ve, so the solution table is optimum.

$z = \frac{38}{5}$, $y_1 = 4$, $y_2 = 9$.

A/2011

Q Solve by Dual Simplex

Minimize: $z = 2x_1 + 2x_2 + 4x_3$

subject to: $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

Sol. Writing the constraints in '≤' form

$$-2x_1 - 3x_2 - 5x_3 \le -2$$

$$3x_1 + x_2 + 7x_3 \le 3$$

$$x_1 + 4x_2 + 6x_3 \le 5 \qquad x_1, x_2, x_3 \ge 0.$$

Writing the LP model in standard form by introducing slack variables.

Min : $z = 2x_1 + 2x_2 + 4x_3$

subject to:

$$-2x_1 - 3x_2 - 5x_3 + s_1 = -2$$

$$3x_1 + x_2 + 7x_3 + s_2 = 3$$

$$x_1 + 4x_2 + 6x_3 + s_3 = 5 \quad , \quad x_1, x_2, x_3, s_1, s_2, s_3 \ge 0.$$

Write the LP model in tableau form

Basic	x_1	x_2 ↓	x_3	s_1	s_2	s_3	sol
z	-2	-2	-4	0	0	0	0
← s_1	-2	$\boxed{-3}$	-5	1	0	0	-2
s_2	3	1	7	0	1	0	3
s_3	1	4	6	0	0	1	5
ratio	$\left\lvert\frac{-2}{-2}\right\rvert = 1$	$\left\lvert\frac{-2}{-3}\right\rvert = \frac{2}{3}$	$\left\lvert\frac{-4}{-5}\right\rvert = \frac{4}{5}$	-	-	-	-

Feasibility ⇒ s_1 leaves.

Optimality ⇒ x_2 enters.

Basic	x_1	x_2	x_3	s_1	s_2	s_3	sol	
z	$-2/3$	0	$-2/3$	$-2/3$	0	0	$4/3$	$R_1 + 2R_2$
x_2	$2/3$	1	$5/3$	$-1/3$	0	0	$2/3$	
s_2	$7/3$	0	$16/3$	$1/3$	1	0	$7/3$	$R_3 - R_2$
s_3	$-1/3$	0	$-2/3$	$4/3$	0	1	$7/3$	$R_4 - 4R_2$

Since no co-efficient in the solution column is −ve, so the table is optimal.

$z = \frac{4}{3}$, $x_2 = \frac{2}{3}$, $x_1 = 0$, $x_3 = 0$.

34

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$-2+3/2$

Basic	x_1	x_2	s_1	s_2	Sol	
z	$-1/2$	0	0	$-3/2$	15	R_1+3R_3
s_1	$1/2$	0	1	$3/2$	15	R_2-3R_3
x_2	$1/2$	1	0	$-1/2$	5	

Since no coefficient in the solution column is -ve.
so the last table is omptimal
Hence $z=15$, $x_1=0$, $x_2=5$.

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Q

Solv

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36 Sol

Su

Q.

Solve Min: $z = 2x_1 + 3x_2$

Subject to.

$$2x_1 + 3x_2 \leq 30$$

$$x_1 + 2x_2 \geq 10$$

$$x_1, x_2 \geq 0.$$

Sol. 37

Writing the constraints in '$\leq$' form

$$2x_1 + 3x_2 \leq 30$$

$$-x_1 - 2x_2 \leq -10$$

$$x_1, x_2 \geq 0.$$

Write the LP model in standard form by introducing slack variable.

Min. $z = 2x_1 + 3x_2$

Subject to: $2x_1 + 3x_2 + S_1 = 30$

$$-x_1 - 2x_2 + S_2 = -10.$$

$$x_1, x_2, S_1, S_2 \geq 0.$$

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Tableau form:

Basic	x_1	x_2 ↓	s_1	s_2	sol.
z	-2	-3	0	0	0
s_1	2	3	1	0	30
← s_2	-1	-2	0	1	-10
ratio	$\left\lvert\frac{-2}{-1}\right\rvert = 2$	$\left\lvert\frac{-3}{-2}\right\rvert = \frac{3}{2}$			

38 Feasibility condition $\Rightarrow$ S_2 leaves

Optimality condition $\Rightarrow$ x_2 enters

Basic	x_1	x_2	s_1	s_2	sol.	
z	0	-1	0	-2	20	$R_1 + 2R_3$
s_1	0	1	1	2	10	$R_2 - 2R_2$
x_2	1	1	0	-1	10	

M-technique.

A/2015.

Q Consider the following LP Model.

Minimize: $z = 2x_1 + 3x_2 - 5x_3$

Subject to: $x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 \geq 10.$

$x_1, x_2, x_3 \geq 0.$

Solve by M-technique.

Sol. Write the model in standard form.

Min. $z = 2x_1 + 3x_2 - 5x_3$

Subject to $x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 - S_1 = 10$

$x_1, x_2, x_3, S_1 \geq 0$

Add non-negative artificial variable R_1 and R_2 to the constraints and M, $M > 0$, per penalty to these artificial variables in the objective function.

Min : $z = 2x_1 + 3x_2 - 5x_3 + MR_1 + MR_2 \rightarrow (1)$

subject to

$x_1 + x_2 + x_3 + R_1 = 7 \rightarrow (2)$

$2x_1 - 5x_2 + x_3 - s_1 + R_2 = 10 \rightarrow (3)$

From (2) and (3) put the values of R_1 and R_2 in (1).

$z = 2x_1 + 3x_2 - 5x_3 + M(7 - x_1 - x_2 - x_3) + M(10 - 2x_1 + 5x_2 - x_3 + s_1)$

$z = (2-3M)x_1 + (3+3M)x_2 - (5+2M)x_3 + ~~17M~~ + Ms_1 + 17M.$

So Then R_1, R_2 are basic variables.

Basic	x_1 ↓	x_2	x_3	s_1	R_1	R_2	sol	ratio
z	$-2+3M$	$-3+3M$	$+5+2M$	$-M$	0	0	$17M$	—
R_1	1	1	1	0	1	0	7	$7/1 = 7$
← R_2	2	-5	1	-1	0	1	10	$10/2 = 5$

Optimality condition ⇒ x_1 is entering.

Feasibility condition ⇒ R_2 is leaving

$R_1 + (2-3M)R_3$

Basic	x_1	x_2 ↓	x_3	S_1	R_1	R_2	sol	ratio	
Z	0	$-8+\frac{9}{2}M$	$6+\frac{1}{2}M$	$-1+\frac{1}{2}M$	0	$1-\frac{3}{2}M$	$10+2M$	—	$R_1-(-2+3M)R_3$
← R_1	0	$\boxed{7/2}$	$1/2$	$1/2$	1	$-1/2$	2	$2\times\frac{2}{7}=\frac{4}{7}$	$R_2 \cdot R_3$
x_1	1	$-5/2$	$1/2$	$-1/2$	0	$1/2$	5	—	

Optimality condition implies that x_2 is entering
Feasibility condition implies that R_1 is leaving

Basic	x_1	x_2	x_3	S_1	R_1	R_2	sol	
Z	0	0	$\frac{50}{7}-\frac{1}{7}M$	$\frac{1}{7}-\frac{1}{7}M$	$\frac{16}{7}-\frac{9}{7}M$	$\frac{-1}{7}-\frac{6}{7}M$	$\frac{102}{7}-\frac{4}{7}M$ ✓	$R_1+\frac{(8-9M)}{2}R_3$
x_2	0	1	$1/7$	$1/7$	$2/7$	$-1/7$	$4/7$	
x_1	1	0	$6/7$	$-1/7$	$5/7$	$1/7$	$45/7$	$R_3+\frac{5}{2}R_2$

Since no coefficient in the objective row is most +ve. Hence the table is ~~optimum~~ feasible

$Z = \frac{102}{7} - \frac{4}{7}M$, $x_1 = \frac{45}{7}$, $x_2 = \frac{4}{7}$, $x_3 = 0$

$$z = 2x_1 + 3x_2 - 5x_3$$

$$\frac{102}{7} - \frac{4M}{7} = 2\left(\frac{45}{7}\right) + 3\left(\frac{4}{7}\right) - 0$$

$$102 - 4M = 90 + 12$$

$$\boxed{M = 0}$$

Thus $z = \frac{102}{7}$, $x_1 = \frac{45}{7}$, $x_2 = \frac{4}{7}$, $x_3 = 0$.

A/o 12.

Q Use Big-M method to solve the following problem.

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to.

$$x_1 + 2x_2 + 3x_3 = 15$$

$$2x_1 + x_2 + 5x_3 = 20$$

$$x_1 + 2x_2 + x_3 + x_4 = 10$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

Sol.

M

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Sol: Write the model in standard form

Max. $z = x_1 + 2x_2 + 3x_3 - x_4$

subject to

$$x_1 + 2x_2 + 3x_3 = 15$$

$$2x_1 + x_2 + 5x_3 = 20$$

$$x_1 + 2x_2 + x_3 + x_4 = 10$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

Mistake

Add non-negative artificial variable R_1, R_2 and R_3 to the contraints and $-M$, $M>0$, per penalty to these artificial variables in the objective function.

$$\text{Max } z = x_1 + 2x_2 + 3x_3 - x_4 - MR_1 - MR_2 - MR_3 \rightarrow (1)$$

Subject to:

$$x_1 + 2x_2 + 3x_3 + R_1 = 15 \rightarrow (2)$$

$$2x_1 + x_2 + 5x_3 + R_2 = 20 \rightarrow (3)$$

$$x_1 + 2x_2 + x_3 + x_4 + R_3 = 10 \rightarrow (4)$$

$$x_1, x_2, x_3, x_4, R_1, R_2, R_3 \geq 0.$$

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Thus R_1, R_2, R_3 are basic variables.

~~Basic x_1 x_2 x_3~~

From, eq (2), (3) and (4) put values of R_1, R_2 and R_3, respectively and put in eq (1)

~~Max~~ $Z = x_1 + 2x_2 + 3x_3 - x_4 - M(15 - x_1 - 2x_2 - 3x_3) - M(20 - 2x_1 - x_2 - 5x_3) - M(10 - x_1 - 2x_2 - x_3 - x_4)$

$Z = (1+4M)x_1 + (2+5M)x_2 + (3+8M) + (-1+M)x_4$

Basic	x_1	x_2	x_3 ↓	x_4	R_1	R_2	R_3	sol	ratio
Z	-1-4M	-2-5M	-3-8M	1-M	0	0	0	0	–
R_1	1	2	3	0	1	0	0	15	15/3 = 5
← R_2	2	1	[5]	0	0	1	0	20	20/5 = 4
R_3	1	2	1	1	0	0	1	10	10/1 = 10

Optimality condition implies x_3 is entering variable.
Feasibility condition implies R_2 is leaving variable.

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Basic	x_1	x_2 ↓	x_3	x_4	R_1	R_2	R_3	sol	ratio	
Z	$-\frac{2}{5}-\frac{4}{5}M$	$-\frac{7}{5}-\frac{17}{5}M$	0	$1-M$	0	$\frac{3}{5}+\frac{8}{5}M$	0	$12+32M$	–	$R_1+(3-2M)R_3$
← R_1	$-7/5$	[$7/5$]	0	0	1	$3/5$	0	3	$3\times\frac{5}{7}=\frac{15}{7}$	R_2-8R_3
x_3	$2/5$	$11/5$	1	0	0	$1/5$	0	4	$4\times5=20$	
R_3	$3/5$	$9/5$	0	1	0	$-1/5$	1	6	$6\times\frac{5}{9}=\frac{10}{3}$	R_4-8R_3

Optimality condition implies that x_2 is entering
Feasibility condition implies that R_1 is leaving

Basic	x_1 ↓	x_2	x_3	x_4	R_1	R_2	R_3	sol	ratio	
Z	$-\frac{3}{5}-\frac{9}{7}M$	0	0	$1-M$	$1+\frac{17}{7}M$	$\frac{1}{7}M$	0	$15+\frac{275}{7}M$	–	$R_1+(\frac{7}{5}+\frac{17}{5}M)R_2$
x_2	$-1/7$	1	0	0	$5/7$	$-3/7$	0	$15/7$	–	
x_3	$11/25$	0	1	0	$-1/5$	$8/25$	0	$17/5$	$\frac{17}{5}\times\frac{25}{11}=\frac{85}{11}$	$R_3-\frac{1}{5}R_2$
← R_3	[$6/7$]	0	1	1	$-9/7$	~~$4/7$~~	1	$3/5$	$\frac{3}{5}\times\frac{7}{6}=\frac{7}{10}$	$R_4-\frac{9}{5}R_2$

Optimality condition implies x_1 is entering
Feasibility condition implies R_3 is leaving

A/2

Q 8

$\frac{19}{10} + \frac{61}{14}M$

Basic	x_1	x_2	x_3	x_4	R_1	R_2	R_3	Sol.	
Z	0	0	$\frac{7}{10}+\frac{3}{2}M$	$\frac{17}{10}+\frac{1}{2}M$	~~$\frac{49}{10}+\frac{61}{14}M$~~	$\frac{2}{5}+M$	$\frac{7}{10}+\frac{3}{7}M$	$\frac{171}{10}+\frac{2813}{70}M$	$R_1+(\frac{3}{5}+\frac{9}{7}M)R_4$
x_2	0	1	$1/6$	$1/6$	$13/14$	$-1/3$	$1/6$	$157/70$	$R_2+\frac{1}{7}R_4$
x_3	0	0	$73/150$	$\frac{-77}{150}$	$-43/50$	$-46/75$	$-77/150$	$\frac{773}{250}$	$R_3-\frac{11}{25}R_4$
x_1	1	0	$7/6$	$7/6$	~~[illegible]~~ $3/2$	~~[illegible]~~ $2/3$	$7/6$	$7/10$	

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Sol.

$Z = \frac{171}{10} + \frac{2813}{70}M$, $x_1 = \frac{7}{10}$, $x_2 = \frac{157}{70}$, $x_3 = \frac{773}{250}$, $x_4 = 0$

~~$Z = 5x_1 + 3x_2 + 10x_3$~~

$Z = x_1 + 2x_2 + 3x_3 - x_4$

$\frac{171}{10} + \frac{2813}{70}M = \frac{7}{10} + 2 \times \frac{157}{70} + 3 \times \frac{773}{250} - 0$

·'·17

Add

+M

func

A/2010.

Q Solve using Big-M technique.

Maximize : $Z = 12x_1 + 20x_2$.

Subject to : $6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$.

$x_1, x_2 \geq 0$.

Sol. Write the model in standard form

Max. $Z = 12x_1 + 20x_2$.

subject to. $6x_1 + 8x_2 - S_1 = 100$

$7x_1 + 12x_2 - S_2 = 120$.

$x_1, x_2, S_1, S_2 \geq 0$.

Add non-negative artificial variables R_1 and R_2 to the constraints and $-M$, $M > 0$, per penalty to these artificial variables in the objective function.

Max. $Z = 12x_1 + 20x_2 - MR_1 - MR_2$ → ①

Subject to: $6x_1 + 8x_2 - S_1 + R_1 = 100$ → ②

$7x_1 + 12x_2 - S_2 + R_2 = 120$ → ③

From ② and ③ put values R_1 and R_2 in eq ①, gives

$Z = 12x_1 + 20x_2 - M(100 - 6x_1 - 8x_2 + S_1) - M(120 - 7x_1 - 12x_2 + S_2)$

$Z = (12 + 13M)x_1 + (20 + 20M)x_2 - MS_1 - MS_2 - 220M$

Thus R_1 and R_2 are Basic variables

Basic	x_1	↓ x_2	S_1	S_2	R_1	R_2	Sol	ratio.
Z	$-12-13M$	$-20-20M$	M	M	0	0	$-220M$	–
R_1	6	8	−1	0	1	0	100	$\frac{100}{8} = 12.5$
← R_2	7	12	0	−1	0	1	120	$\frac{120}{12} = 10$

Optimality condition implies that x_2 is entering.

Feasibility condition implies that R_2 is leaving

Basic	x_1 ↓	x_2	S_1	S_2	R_1	R_2	sol	ratio.	
z	$-\frac{1}{3}-\frac{4}{3}M$	0	M	$-\frac{5}{3}+\frac{2}{3}M$	0	$\frac{5}{3}+\frac{5}{3}M$	$200-20M$	–	R_1+[illegible]R_3
← R_1	2	0	-1	$\frac{2}{3}$	1	$-\frac{2}{3}$	20	$\frac{20}{2}=10$	R_2-8R_3
x_2	$\frac{7}{12}$	1	0	$-\frac{1}{12}$	0	$\frac{1}{12}$	10	$10\times\frac{12}{7}=\frac{120}{7}$	

Optimality condition implies x_1 is entering.
Feasibility condition implies R_1 is leaving.

Basic	x_1	x_2	S_1	S_2 ↓	R_1	R_2	sol.	ratio	
z	0	0	$-\frac{1}{6}+\frac{1}{3}M$	$-\frac{14}{9}-\frac{2}{9}M$	$\frac{1}{6}+\frac{2}{3}M$	$\frac{14}{9}+\frac{11}{9}M$	$\frac{610}{3}-\frac{20}{3}M$		$R_1+(\frac{1}{3}+\frac{4}{3}M)R_2$
← x_1	1	0	$-\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{3}$	10	30	
x_2	0	1	$\frac{7}{24}$	$-\frac{5}{18}$	$-\frac{7}{12}$	$\frac{5}{18}$	$\frac{25}{6}$	–	$R_3-\frac{7}{12}R_2$

Basic	x_1	x_2	S_1	S_2	R_1	R_2	sol.	
z	$\frac{14}{3}+\frac{2}{3}M$	0	$-\frac{5}{2}$	0	$\frac{5}{2}+M$	M	250	$R_1+(\frac{14}{9}+\frac{2}{9}M)R_2$
S_2	3	0	$-\frac{3}{2}$	1	$\frac{3}{2}$	-1	30	
x_2	$\frac{5}{6}$	1	$\frac{11}{72}$	0	$-\frac{1}{6}$	0	$\frac{25}{2}$	$R_3+\frac{5}{18}R_2$

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Since no coefficient in the objective row is -ve

Hence the solution is

$z = 250, \quad x_1 = 0, \quad x_2 = \frac{25}{2}, \quad S_2 = 30.$

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N

Assignments

Q. Max $z = x_1 + 5x_2 + 3x_3$.

Subject to. $x_1 + 2x_2 + x_3 = 3$

$2x_1 - x_2 = 4$

$x_1, x_2, x_3 \geq 0$.

Solve using x_3 and R for starting Basic solution.

Sol. Write the model in Standard form

Max $z = x_1 + 5x_2 + 3x_3$.

Subject to $x_1 + 2x_2 + x_3 = 3$

$2x_1 - x_2 = 4$

$x_1, x_2, x_3 \geq 0$.

S

Scanned with CamScanner

Add non-negative artificial variable R_1 and R_2 to the constraints and $-M$, $M \geq 0$, per penalty to these artificial variables in the objective function.

Maximize $Z = x_1 + 5x_2 + 3x_3 - MR_1 - MR_2$ → (1)

subject to. $x_1 + 2x_2 + x_3 + R_1 = 3$ → (2)

$2x_1 - x_2 + R_2 = 4$. → (3)

$x_1, x_2, x_3, R_1, R_2 \geq 0$.

From (2) and (3) put values of R_1 and R_2 in (1), we have

$Z = x_1 + 5x_2 + 3x_3 - M(3 - x_1 - 2x_2 - x_3) - M(4 - 2x_1 + x_2)$

$Z = (1 + 3M)x_1 + (5 + M)x_2 + 3x_3 - 7M$.

Thus R_1 and R_2 are basic, but it is given that we start our solution using x_3 and R_2 as starting basic solution

$2 \to \frac{1}{2}$

Basic	x_1 ↓	x_2	x_3	R_1	R_2	sol	ratio
z	$-1-3M$	$-5-M$	-3	0	0	$-7M$	—
x_3	1	2	1	1	0	3	$3/1 = 3$
← R_2	2	-1	0	0	1	4	$4/2 = 2$

Optimality condition implies that x_1 is entering

Feasibility condition implies that R_2 is leaving

So

Basic	x_1	x_2 ↓	x_3	R_1	R_2	sol	ratio	
z	0	$-\frac{11}{2}-\frac{5}{2}M$	-3	0	$\frac{1}{2}+\frac{3}{2}M$	$2-M$		$R_1+(1+3M)R_3$
← x_3	0	$\boxed{5/2}$	1	1	$-1/2$	1	$1 \times \frac{2}{5}$	R_2-R_3
x_1	1	$-1/2$	0	0	$1/2$	2	—	

Optimality ⟹ x_2 is entering, Feasibility implies x_3 is leaving

Basic	x_1	x_2	x_3	R_1	R_2	sol	
z	0	0	$-\frac{4}{5}+M$	$\frac{11}{5}+M$	$-\frac{3}{5}+M$	$21/5$	$R_1+(\frac{11}{2}+\frac{5}{2}M)R_2$
x_2	0	1	$2/5$	$2/5$	$-1/5$	$2/5$	
x_1	1	0	$1/5$	$1/5$	$3/5$	$11/5$	$R_3+\frac{1}{2}R_2$

Since no coefficient in the objective row is negative. Hence the solution is

$$Z = \frac{21}{5}, \quad x_1 = \frac{11}{5} \;\&\; x_2 = \frac{2}{5}$$

Q. Max. $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

subject to. $x_1 + x_2 + x_3 = 4$

$x_1 + 4x_2 + x_4 = 8$. $\quad x_1, x_2, x_3, x_4 \geq 0$

Solve using x_3 and x_4 as starting basic solution

Sol.

~~Let~~ Add non-negative artificial variable to the contraints with $-M, M>0$, per penalty to these artificial variable to the objective function

Max. $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4 - MR_1 - MR_2 \longrightarrow (1)$

subject to. $x_1 + x_2 + x_3 + R_1 = 4 \longrightarrow (2)$

$x_1 + 4x_2 + x_4 + R_2 = 8 \longrightarrow (3) \quad x_1, x_2, x_3, x_4, R_1, R_2 \geq 0.$

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From (2) and (3) put values of R_1 and R_2 in (1), we have

$$Z = 2x_1 + 4x_2 + 4x_3 - 3x_4 - M(4 - x_1 - x_2 - x_3) - M(8 - x_1 - 4x_2 - x_4)$$

$$z = (2+2M)x_1 + (4+5M)x_2 + (4+M)x_3 + (-3+M)x_4 - 12M.$$

Thus R_1 and R_2 are basic variables, but we are given that x_3 and x_4 use for starting basic solution

Basic	x_1	x_2 ↓	x_3	x_4	R_1	R_2	sol	ratio.
z	$-2-2M$	$-4-5M$	$-4-M$	$3-M$	0	0	$-12M$	—
x_3	1	1	1	0	1	0	4	$4/1 = 4$
← x_4	1	[4]	0	1	0	1	8	$8/4 = 2$

Optimality ⇒ x_2 is entering, Feasibility ⇒ x_4 is leaving

Basic	x_1	x_2	x_3	x_4	R_1	R_2	sol	
z	$-1-\frac{3}{4}M$	0	$-4-M$	$4+\frac{1}{4}M$	0	$1+\frac{5}{4}M$	$8-2M$	$R_1+(4+5M)R_3$
x_3	$3/4$	0	1	$-1/4$	1	$-1/4$	2	
x_2	$1/4$	1	0	$1/4$	0	$1/4$	2	

$3/4 - 1/4$

Basic	x_1	x_2	x_3	x_4	R_1	R_2	sol
z	2	0	0	-3	4+M	M	16
x_3	3/4	0	1	-1/4	1	-1/4	2
x_2	1/4	1	0	1/4	0	1/4	2.

$R_1+(4+M)$

Since no co-efficient in the objective row is negative
~~Hence~~ So the solution is

$z = 16, \ x_4 = 0, \ x_2 = 2, \ x_3 = 2, \ x_1 = 0$

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Q. Solve using x_3 and x_4 as starting Basic solution.

Min: $z = 3x_1 + 2x_2 + 3x_3$

subject to: $x_1 + 4x_2 + 3x_3 \geq 7$

$2x_1 + x_2 + x_4 \geq 10$

$x_1, x_2, x_3, x_4 \geq 0.$

$+(2+5M)R_3$

Write the LP model in standard form

Min $z = 3x_1 + 2x_2 + 3x_3$

Subject to

$$x_1 + 4x_2 + 3x_3 - S_1 = 7$$

$$2x_1 + x_2 + x_4 + S_2 = 10$$

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$$x_1, x_2, x_3, x_4, S_1, S_2 \geq 0$$

Add non-negative artificial variable R_1 and R_1 to the constraints and M, $M>0$, per penalty to these artificial variables in the objective function.

Min : $Z = 3x_1 + 2x_2 + 3x_3 + MR_1 + MR_2 \rightarrow (1)$

Subject to: $x_1 + 4x_2 + 3x_3 - S_1 + R_1 = 7 \rightarrow (2)$

$x_1 + x_2 + x_4 - S_2 + R_2 = 10 \rightarrow (3)$

$$x_1, x_2, x_3, x_4, S_1, S_2, R_1, R_2 \geq 0$$

From (2) and (3) put the values of R_1 and R_2 in (1), we have

$z = 3x_1 + 2x_2 + 3x_3 + M(7 - x_1 - 4x_2 - 3x_3 + S_1) + M(10 - x_1 - x_2 - x_4 + S_2)$

$z = (3-2M)x_1 + (2-5M)x_2 + (3-3M)x_3 - Mx_4 + MS_1 + MS_2 + 17M$

Basic	x_1	x_2 ↓	x_3	x_4	S_1	S_2	R_1	R_2	sol	ratio.
Z	$-3+2M$	$-2+5M$	$-3+3M$	M	$-M$	$-M$	0	0	$17M$	–
← x_3	1	[4]	3	0	-1	0	1	0	7	$7/4$
x_4	1	1	0	1	0	-1	0	1	10	$10/1$

Optimality condition implies x_2 is entering.
Feasibility condition implies x_3 is leaving.

Basic	x_1 ↓	x_2	x_3	x_4	S_1	S_2	R_1	R_2	sol	ratio.	note
Z	$-\frac{5}{2}+\frac{3}{4}M$	0	$-\frac{3}{2}\,\frac{3}{4}M$	M	$-\frac{1}{2}+\frac{1}{4}M$	$-M$	$\frac{1}{2}-\frac{5}{4}M$	0	$\frac{7}{2}+\frac{33}{4}M$		$R_1+(2-5M)R_2$
x_2	[1/4]	1	$3/4$	0	$-1/4$	0	$1/4$	0	$7/4$	$\frac{7}{4}\times 4=7$	
x_4	$3/4$	0	$-3/4$	1	$1/4$	-1	$-1/4$	1	$33/4$	$\frac{33}{4}\times\frac{4}{3}=11$	R_3-R_2

Optimality ⇒ x_1 enters, ~~Feasibility implies x_2 leaves~~

Basic	x_1	x_2	x_3	x_4	S_1	S_2	R_1	R_2	sol	ratio.	note
Z	0	$10-3M$	$6-3M$	M							$R_1+(\frac{5}{2}-\frac{3}{4}M)R_2$
x_1	4	4	3	0	-1	0	1	0	7		
x_4	0	-3	-3	1	1	-1	-1	1	3		$R_3-\frac{3}{4}R_2$

Basic	x_1	x_2	x_3	x_4	S_1	S_2	R_1	R_2	sol	
z	$-\frac{5}{2}$	0	$-3/2$	0	$-1/2$	0	$1/2-M$	$-M$	$7/2$	$R_1 - MR_2$
x_2	$1/4$	1	$3/4$	0	$-1/4$	0	$1/4$	0	$7/4$	
x_4	$3/4$	0	$-3/4$	1	$1/4$	-1	$-1/4$	0	$33/4$	

Since no coefficient in the objective row is most +ve

So the solution is

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$z = 7/2$, $x_1 = 0$, $x_2 = 7/4$, $x_3 = 0$, $x_4 = \frac{33}{4}$

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Two-phase Method.

A/2,017

Q2.(b) Solve the following by using two phase method.

Min: $z = 2x_1 + 3x_2 - 5x_3$ → (A)

Subject to.

$x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 \geq 10$. , $x_1, x_2, x_3 \geq 0$.

Sol. Write the LP model in standard form

Min : $z = 2x_1 + 3x_2 - 5x_3$

subject to $x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 - S_1 = 10$, $x_1, x_2, x_3, S_1 \geq 0$.

Add artificial non-negative variable to the constraints

$x_1 + x_2 + x_3 + R_1 = 7$ → (1)

$2x_1 - 5x_2 + x_3 - S_1 + R_2 = 10$ → (2)

Phase I.

Min : $r_0 = R_1 + R_2$ → (3)

Befores putting the problem in tableau form, we express r_0 in terms of non-basic variables. For this purpose put the

values of R_1 and R_2 from (1) and (2) in (3)

$r_0 = 7 - x_1 - x_2 - x_3 + 10 - 2x_1 + 5x_2 - x_3 + s_1$

$r_0 = -3x_1 + 4x_2 - 2x_3 + s_1 + 17$

$r_0 + 3x_1 - 4x_2 + 2x_3 - s_1 = 17$

Tableau form:

Basic	x_1 ↓	x_2	x_3	s_1	R_1	R_2	sol	ratio
r_0	3	-4	2	-1	0	0	17	—
R_1	1	1	1	0	1	0	7	$7/1 = 7$
← R_2	2	-5	1	-1	0	1	10	$10/2 = 5$

Optimality implies x_1 enters ; Feasibility implies R_2 leaves

Basic	x_1	x_2 ↓	x_3	s_1	R_1	R_2	sol	ratio	
r_0	0	$7/2$	$1/2$	$1/2$	0	$-3/2$	2	—	$R_1 - 3R_3$
← R_1	0	[$7/2$]	$1/2$	$1/2$	1	$1/2$	2	$2 \times \frac{2}{7} = 4/7$	$R_1 - R_3$
x_1	1	$-5/2$	$1/2$	$-1/2$	0	$1/2$	5	—	

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Basic	x_1	x_2	x_3	s_1	R_1	R_2	sol	ratio
r_0	0	0	0	0	-1	-2	0	$R_1 - \frac{7}{2}R_1$
x_2	0	1	1/7	1/7	2/7	1/7	4/7	
x_1	1	0	6/7	-1/7	5/7	6/7	45/7	$R_3 + \frac{5}{2}R_2$

Since $r_0 = 0$ in the optimum table so the problem has feasible solution and we move toward phase 2

Phase 2.

From the optimum table

$$x_2 + \frac{1}{7}x_3 + \frac{1}{7}s_1 = \frac{4}{7} \Rightarrow x_2 = \frac{4}{7} - \frac{1}{7}x_3 - \frac{1}{7}s_1 \rightarrow (4)$$

$$x_1 + \frac{6}{7}x_3 - \frac{1}{7}s_1 = \frac{45}{7} \Rightarrow x_1 = \frac{45}{7} - \frac{6}{7}x_3 + \frac{1}{7}s_1 \rightarrow (5)$$

$R_1 - 3R_3$

$- R_3$

Put (4) and (5) in (A)

$$Z = 2\left(\frac{45}{7} - \frac{6}{7}x_3 + \frac{1}{7}s_1\right) + 3\left(\frac{4}{7} - \frac{1}{7}x_3 - \frac{1}{7}s_1\right) - 5x_3$$

$$z = -\frac{50}{7}x_3 - \frac{1}{7}s_1 + \frac{102}{7}$$

$$z + \frac{50}{7}x_3 + \frac{1}{7}s_1 = \frac{102}{7}$$

Basic	x_1	x_2	x_3 ↓	s_1	Sol	ratio.
z	0	0	$50/7$	$1/7$	$102/7$	
← x_2	0	1	$\boxed{1/7}$	$1/7$	$4/7$	$4/7 \times 7 = 4$
x_1	1	0	$6/7$	$-1/7$	$45/7$	$45/7 \times 7/6 = \frac{15}{2}$

Optimality implies x_3 enters, Feasibility implies x_2 leaves

Basic	x_1	x_2	x_3	s_1	sol	ratio.	
z	0	-50	0	$-349/7$	-14.		$R_1 - \frac{50}{7}R_2$
x_3	0	7	1	7	4		
x_1	1	$-6/7$	0	$-43/7$	3		$R_3 - \frac{6}{7}R_2$

Since no coefficient in the objective row is +ve

So the solution is $z = -14$, $x_1 = 3$, $x_2 = 0$, $x_3 = 4$.

A/2011

Q2(b) Solve the following problem by using Two-phase method.

Minimize : $Z = \frac{15}{2} x_1 - 3x_2$ → (f)

Subject to: $3x_1 - x_2 - x_3 \geq 3$

$x_1 - x_2 + x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$.

Sol. Write the constraints in standard form and add artificial variables R_1 and R_2 to the constraints

$3x_1 - x_2 - x_3 - S_1 + R_1 = 3$ → (1)

$x_1 - x_2 + x_3 - S_2 + R_1 = 2$ → (2)

$x_1, x_2, S_1, S_2, R_1, R_2 \geq 0$

Phase 1.

Min : $r_0 = R_1 + R_2$ → (3)

put the values of R_1 and R_2 from (1) and (2) in (3)

$r_0 = 3 - 3x_1 + x_2 + x_3 + S_1 + 2 - x_1 + x_2 - x_3 + S_2$

$r_0 = -4x_1 + 2x_2 + 0x_3 + S_1 + S_2 + 5$

Basic	x_1	x_2	x_3	s_1	s_2	R_1	R_2	sol	ratio.
r_0	4	-2	0	-1	-1	0	0	5	—
← R_1	[3]	-1	-1	-1	0	1	0	3	$3/3 = 1$
R_2	1	-1	1	0	-1	0	1	2	$2/1 = 2$

optimality implies x_1 enters, Feasibility implies R_1 leaves

Basic	x_1	x_2	x_3	s_1	s_2	R_1	R_2	sol	ratio.	
r_0	0	$-2/3$	$4/3$	$1/3$	-1	$-4/3$	0	1		$R_1 - 4R_2$
x_1	1	$-1/3$	$-1/3$	$-1/3$	0	$1/3$	0	1	-	
← R_2	0	$-2/3$	[$4/3$]	$1/3$	-1	$-1/3$	1	1	$1 \times 3/4 = 3/4$	$R_3 - R_2$

optimality implies x_3 enters, Feasibility implies R_2 leaves

Basic	x_1	x_2	x_3	s_1	s_2	R_1	R_2	sol	
r_0	0	0	0	0	0	-1	-1	0	$R_1 - \frac{4}{3} R_3$
x_1	1	$-1/2$	0	$-1/4$	$-1/4$	$1/4$	$1/4$	$5/4$	$R_2 + \frac{1}{3} R_2$
x_3	0	$-1/2$	1	$1/4$	$-3/4$	$-1/4$	$3/4$	$3/4$	

Since $\gamma_0 = 0$ in the optimum table so the problem has feasible solution and we move towards phase 2.

Phase 2.

From the optimum table

$$x_1 - \frac{1}{2}x_2 - \frac{1}{4}s_1 - \frac{1}{4}s_2 = \frac{5}{4} \Rightarrow x_1 = \frac{1}{2}x_2 + \frac{1}{4}s_1 + \frac{1}{4}s_2 + \frac{5}{4} \rightarrow (4)$$

$$-\frac{1}{2}x_2 + x_3 + \frac{1}{4}s_1 - \frac{3}{4}s_2 = \frac{3}{4} \Rightarrow x_3 = \frac{1}{2}x_2 - \frac{1}{4}s_1 + \frac{3}{4}s_2 + \frac{3}{4} \rightarrow (5)$$

$R_1 - 4R_3$

Put (4) and (5) in (A)

$R_3 - R_2$

$$z = \frac{15}{2}\left(\frac{1}{2}x_2 + \frac{1}{4}s_1 + \frac{1}{4}s_2 + \frac{5}{4}\right) - 3x_2$$

$$z = \frac{7}{4}x_2 + \frac{15}{8}s_1 + \frac{15}{8}s_2 + \frac{75}{8}$$

$R_1 - \frac{4}{3}R_3$

$$z - \frac{7}{4}x_2 - \frac{15}{8}s_1 - \frac{15}{8}s_2 = \frac{75}{8}$$

$R_2 + \frac{1}{3}R_2$

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Basic	x_1	x_2	x_3	s_1	s_2	Sol
z	0	$-7/4$	0	$-15/8$	$-15/8$	$75/8$
x_1	1	$-1/2$	0	$-1/4$	$-1/4$	$5/4$
x_3	0	$-1/2$	1	$1/4$	$-3/4$	$3/4$

Since no coefficient in the objective row is +ve.

So the solution is.

$$z = \frac{75}{8}, \quad x_1 = \frac{5}{4}, \quad x_2 = 0, \quad x_3 = \frac{3}{4}$$

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Duality and Post Optimal Analysis

A/2017

Q3) Write the dual of the following problem, primal LP and determine the values of dual variables by solving primal LP model.

$$\text{Max}: \; z = 15x_1 + 12x_2$$

Subject to:

$$x_1 + 2x_2 \geq 3$$

$$2x_1 - 4x_2 \leq 5.$$

$$x_1, x_2 \geq 0.$$

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Write the LP model in standard form.

$$\text{Max}: \; z = 15x_1 + 12x_2 - MR_1 \rightarrow (1)$$

subject to:

$$x_1 + 2x_2 - s_1 + R_1 = 3. \rightarrow (2)$$

$$2x_1 - 4x_2 + s_2 = 5 \rightarrow (3)$$

Its dual is.

$$\text{Min} \; w = 3y_1 + 5y_2$$

subject to

$$y_1 + 2y_2 \geq 15$$

$$2y_1 - 4y_2 \geq 12$$

$-y_1 \geq 0 \Rightarrow y_1 \leq 0.$

$y_1 \geq -M$ $\quad y_1$ is unrestricted

$y_2 \geq 0$

Solution of primal LP model

From ② put value of R_1 in ①

$Z = 15x_1 + 12x_2 - M(3 - x_1 - 2x_2 + s_1)$

$Z = (15+M)x_1 + (12+2M)x_2 - Ms_1 - 3M$

Basic	x_1	x_2 ↓	s_1	s_2	R_1	sol.	ratio.
Z	$-15-M$	$-12-2M$	0	M	0	$-3M$	—
← R_1	1	2	-1	0	1	3	$3/2$
s_2	2	-4	0	1	0	5	—

Optimality ⇒ x_1 enters

Feasibility ⇒ R_1 leaves

Basic	x_1	x_2	s_1	s_2	R_1	sol.	
z	-9	0	$-6-M$	M	$6+M$	18	$R_1+(12+2M)R_2$
x_2	$1/2$	1	$-1/2$	0	$1/2$	$3/2$	
s_2	4	0	-2	1	2	11	R_2+4R_2

Basic	x_1 ↓	x_2	s_1	s_2	R_1	sol.	ratio.	
z	$-9-4M$	0	$-6+M$	0	$6-M$	$18-11M$		R_1-MR_3
x_2	$1/2$	1	$-1/2$	0	$1/2$	$3/2$	$\frac{3}{2}\times 2 = 3$	
← s_2	$\boxed{4}$	0	-2	1	2	11	$\frac{11}{4} =$	

Optimality ⇒ x_1 enters, Feasibility implies s_2 leaves

Basic	x_1	x_2	s_1	s_2	R_1	sol.	
z	0	0	$-\frac{21}{2}-M$	$\frac{9}{4}+M$	$\frac{21}{2}+M$	$\frac{171}{4}$	$R_1+(9+4M)R_3$
x_2	0	1	$-1/4$	$-1/8$	$1/4$	$1/8$	$R_2-\frac{1}{2}R_3$
x_1	1	0	$-1/2$	$1/4$	$1/2$	$11/4$	

Hence the solution is $z = \frac{171}{4}$, $x_1 = \frac{11}{4}$, $x_2 = \frac{1}{8}$

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$$\text{Optimal inverse} = \begin{pmatrix} -1/8 & 1/4 \\ 1/4 & 1/2 \end{pmatrix}.$$

Thus, the optimal dual values are computed, as

$$(y_1, y_2) = \begin{pmatrix} \text{original objective} \\ \text{coefficients of } x_2, x_1 \end{pmatrix} \times \begin{pmatrix} \text{Optimal inverse} \end{pmatrix}.$$

$$(12, 15) \begin{pmatrix} -1/8 & 1/4 \\ 1/4 & 1/2 \end{pmatrix}$$

$$(y_2, y_1) \quad \left(\frac{9}{4}, \frac{21}{2}\right)$$

Hence $y_1 = \frac{21}{2}$, $y_2 = \frac{9}{4}$; $w = \frac{171}{4}$

~~[illegible]~~ A/2013.

Q3(a) Use M-technique to solve the following

Minimize $z = 5x_1 + 12x_2 + 4x_3$

Subject to $x_1 + 2x_2 + x_3 \leq 10$

$2x_1 - x_2 + 3x_3 \leq 8$

$x_1, x_2, x_3 \geq 0$.

b) Write the dual of the above problem. Find its optimal dual solution using the optimal solution of part (a)

Sol. Write the model in standard form

Min: $z - 5x_1 + 12x_2 + 4x_3$

Subject to: $x_1 + 2x_2 + x_3 + s_1 = 10$

$2x_1 - x_2 + 3x_3 + s_2 = 8$.

Basic	x_1	x_2	x_3	s_1	s_2	sol.
z	-5	-12	-4	0	0	-
s_1	1	2	1	1	0	10
s_2	2	-1	3	0	1	8

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A/2012(New)

Q no 3 Write the dual corresponding to

Maximize $z = 5x_1 + 8x_2 + 10x_3$

Subject to $x_1 + x_2 + 2x_3 \leq 120$

$3x_1 - 2x_2 - x_3 \geq 90$

$2x_1 + 4x_2 + 2x_3 = 100$

$x_1, x_2, x_3 \geq 0.$

Sol. Write the primal LP in standard form

Maximize $z = 5x_1 + 8x_2 + 10x_3 + 0S_1 + 0S_2 + 0S_3 - MR_1 - MR_2$

Subject to $x_1 + x_2 + 2x_3 + S_1 = 120$

$3x_1 - 2x_2 - x_3 - S_2 + R_1 = 90$

$2x_1 + 4x_2 + 2x_3 - S_3 + R_2 = 100$

$x_1, x_2, x_3, S_1, S_2, S_3, R_1, R_2 \geq 0$

It's Dual is

Minimize $w = 120y_1 + 90y_2 + 100y_3$

Subject to $y_1 + 3y_2 + 2y_3 \geq 5$

$y_1 - 2y_2 + 4y_3 \geq 8.$

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$2y_1 - y_2 + 2y_3 \geq 10$

$y_1 \geq 0$

$-y_1 \geq 0 \Rightarrow y_2 \leq 0$

$-y_3 \geq 0 \Rightarrow y_3 \leq 0$

$y_2 \geq -M \Rightarrow y_2$ is unrestricted

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Assignments.

Q. Max. $z = x_1 + 2x_2 + x_3 \rightarrow$ (A)

subject to: $x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 \geq 10$

$x_1, x_2, x_3 \geq 0.$

Sol.

Write the constraints in the standard form and introduce artificial variables R_1 and R_2 to the constraints

$x_1 + x_2 + x_3 + R_1 = 7 \longrightarrow$ (1)

$2x_1 - 5x_2 + x_3 - S_1 + R_2 = 10. \rightarrow$ (2)

Phase 1.

Max. $r_0 = R_1 + R_2 \rightarrow$ (3).

Put values of R_1 and R_2 in (3)

$r_0 = 7 - x_1 - x_2 - x_3 + 10 - 2x_1 + 5x_2 - x_3 + S_1$

$r_0 = -3x_1 + 4x_2 - 2x_3 + S_1 + 17.$

$r_0 + 3x_1 - 4x_2 + 2x_3 - S_1 = 17$

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Basic	x_1	x_2 ↓	x_3	s_1	R_1	R_2	sol	ratio
r_0	3	-4	2	-1	0	0	17	—
← R_1	1	1	1	0	1	0	7	$7/1 = 7$
R_2	2	-5	1	-1	0	1	10	$10/-5 = -2$

Optimality ⇒ x_2 is entering, Feasibility ⇒ R_1 is leaving

Basic	x_1	x_2	x_3	s_1 ↓	R_1	R_2	sol	ratio	
r_0	7	0	6	-1	4	0	45	—	$R_1 + 4R_2$
x_2	1	1	1	0	1	0	7	—	
R_2	7	0	6	-1	5	1	45	—	$R_2 + 5R_1$

Q. Min. $Z = 2x_1 + 3x_2 - 5x_3 \rightarrow$ (A)

Subject to: $x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 \geq 10$. $x_1, x_2, x_3 \geq 0$.

Sol.

Write the constraints in standard form and add non-negative artificial variable to the constraints:

$$x_1 + x_2 + x_3 + R_1 = 7 \rightarrow (1)$$

$$2x_1 - 5x_2 + x_3 - S_1 + R_2 = 10 \rightarrow (2)$$

$$x_1, x_2, x_3, S_1, R_1, R_2 \geq 0$$

Phase 1.

$$\text{Min } r_0 = R_1 + R_2 \rightarrow (3)$$

Put values of R_1 and R_2 from (1) and (2) in (3), we have

$$r_0 = 7 - x_1 - x_2 - x_3 + 10 - 2x_1 + 5x_2 - x_3 + S_1$$

$$r_0 = -3x_1 + 4x_2 - 2x_3 + S_1 + 17$$

$$r_0 + 3x_1 - 4x_2 + 2x_3 - S_1 = 17.$$

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Basic	x_1 ↓	x_2	x_3	s_1	R_1	R_2	sol	ratio
r_0	3	-4	2	-1	0	0	17	—
R_1	1	1	1	0	1	0	7	$7/1 = 7$
← R_2	[2]	-5	1	-1	0	1	10	$10/2 = 5$

Optimality ⇒ x_1 is ~~leaving~~ entering , Feasibility ⇒ R_2 is leaving

Basic	x_1	x_2	x_3 ↓	s_1	R_1	R_2	sol	ratio	
r_0	0	$-23/2$	$1/2$	$1/2$	0	$-3/2$	2		$R_1 \pm 3R_3$
← R_1	0	$7/2$	[$1/2$]	$1/2$	1	$-1/2$	2	$2\times2=4$	$R_2 - R_3$
x_1	1	$-5/2$	$1/2$	$-1/2$	0	$1/2$	5	$5\times2=10$	

Basic	x_1	x_2	x_3	s_1	R_1	R_2	sol	
r_0	0	$-53/4$	0	0	-1	-1	0	$R_1 - \frac{1}{3}R_2$
x_3	0	7	[1]	1	2	-1	4	
x_1	1	-6	0	-1	-1	1	4	$R_3 - \frac{1}{2}R_2$

Since $r_0 = 0$, so the problem has feasible solution, we move towards Phase 2

Phase 2.

From last optimum table

$$7x_2 + x_3 + s_1 = 4 \Rightarrow x_3 = 4 - 7x_2 - s_1 \rightarrow (4)$$

$$x_1 - 6x_2 - s_1 = 4 \Rightarrow x_1 = 6x_2 + s_1 + 4 \rightarrow (5)$$

Put (4) and (5) in (A)

$$z = 2(6x_2 + s_1 + 4) + 3x_2 - 5(-7x_2 - s_1 + 4)$$

$$z = -23x_2 - 3s_1 - 12$$

$$z + 23x_2 + 3s_1 = -12$$

Basic	x_1	x_2 ↓	x_3	s_1	sol	ratio.
z	0	23	0	3	-12	–
← x_3	0	[7]	1	1	4	4/7
x_1	1	-6	0	-1	4	–

Optimality ⇒ x_2 enters. Feasibility ⇒ x_3 leaves

Basic	x_1	x_2	x_3	s_1	sol	
z	0	0	$-23/7$	$-2/7$	$-176/7$	$R_1 - 23R_2$
x_2	0	1	$1/7$	$1/7$	$4/7$	
x_1	1	0	$6/7$	$-1/7$	$1/7$	$R_3 + 6R_2$

∞

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Method of Multipliers.

A/2016

Q no 4 (b) Apply the Method of Multipliers to solve the following model using initial basic feasible solution by Vogel's Approximation method. (VAM)

			Supply
1	2	6	7
0	4	2	12
3	1	5	11
Demand 10	10	10	

Sol Step I. Basic feasible solution.

First we find basic feasible solution using Vogel Approx method.

				Row penalty
1	2	6	7	2-1 = 1
0	4	10 2	~~12~~ = 2	2-0 = 2
3	1	5	11	3-1 = 2
10	10	~~10~~ 0		
Column penalty 1-0 = 1	2-1 = 1	5-2 = 3		

				row penalty
1	2	7		2-1 = 1
2 (0)	4	~~2~~ 0		4-0 = 4
3	1	11		3-1 = 2
~~10~~ 8	10			
1-0=1	2-1=1		column penalty	

			row penalty
1	2	7	2-1 = 1
3	10 (1)	~~11~~ 1	3-1 = 2
8	~~10~~ 0		
3-1=2	2-1=1	column penalty	

		row penalty
1	7	1
1 (3)	~~1~~ 0	3
~~8~~ 7		
3-1=2	column penalty	

7 [1]

7/.

7
0

Hence the starting Basic feasible solution is

	$v_1 = 1$	$v_2 = -1$	$v_3 = 3$	
$u_1 = 0$	7 [1]	[2]	[6]	7
$u_2 = -1$	2 [0]	[4]	10 [2]	12
$u_3 = 2$	1 [3]	10 [1]	[5]	11
	10	10	10	

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Step II. Method of Multipliers.

Basic variables	(u,v) equations	solution
c_{11}	$u_1 + v_1 = 1$	$u_1 = 0 \Rightarrow v_1 = 1$
c_{21}	$u_2 + v_1 = 0$	$u_2 = -1,\ v_1 = 1$
c_{23}	$u_2 + v_3 = 2$	$u_2 = -1,\ v_3 = 3$
c_{31}	$u_3 + v_1 = 3$	$u_3 = 2,\ v_1 = 1$
c_{32}	$u_3 + v_2 = 1$	$u_3 = 2,\ v_2 = -1$

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Non-basic variables $\quad \bar{c}_{ij} = u_i + v_j - c_{ij}$

$\bar{c}_{12}$	$u_1 + v_2 - c_{12} =$	$0 + (-1) - 2 = -3$
$\bar{c}_{13}$	$u_1 + v_3 - c_{13} =$	$0 + 3 - 6 = -3$
$\bar{c}_{22}$	$u_2 + v_2 - c_{22} =$	$-1 - 1 - 4 = -6$
$\bar{c}_{33}$	$u_3 + v_3 - c_{33} =$	$2 + 3 - 5 = 0$

$\therefore$ Since none of the $\bar{c}_{ij}$'s are most +ve. Hence the last table is optimal

Total cost $= 7 \times 1 + 2 \times 0 + 10 \times 2 + 1 \times 3 + 10 \times 1 = 40$

A/2015

Qno 5(a). Solve the following transportation model using method of multipliers with northwest corner method as starting solution.

				Supply
	0	2	1	5
	2	1	5	10
	2	4	3	5
Demand	5	5	10	

Sol. Step I. Starting feasible solution

North west corner method

				supply
	5 (0)	(2)	(1)	5 0
	(2)	5 (1)	5 (5)	10 5 0
	(2)	(4)	5 (3)	5 0
demand	5 0	5 0	10 5	

The starting basic feasible solution is

5 (0)	(2)	(1)
(2)	5 (1)	5 (5)
(2)	(4)	5 (3)

Step 2. Method of multipliers.

	$v_1=0$	$v_2=0$	$v_3=4$
$u_1=0$	[−] 0 / 5	2	1 [+]
$u_2=1$	[+] 2	1 / 5	5 / 5 [−]
$u_3=-1$	2	4	3 / 5

Basic variables c_{ij}	(u, v) equation $c_{ij} = u_i + v_j$	solution
c_{11}	$u_1 + v_1 = 0$	$u_1 = 0$, $v_1 = 0$
c_{22}	$u_2 + v_2 = 1$	$u_2 = 1$, $v_2 = 0$
c_{23}	$u_2 + v_3 = 5$	$u_2 = 1$, $v_3 = 4$
c_{33}	$u_3 + v_3 = 3$	$u_3 = -1$, $v_3 = 4$

Non basic variables $\bar{c}_{ij}$	$\bar{c}_{ij} = u_i + v_j - c_{ij}$	
$\bar{c}_{12}$	$u_1 + v_2 - c_{12} = 0 + 0 - 2 = -2$	
$\bar{c}_{13}$	$u_1 + v_3 - c_{13} = 0 + 4 - 1 = 3$	(Entering variable)
$\bar{c}_{21}$	$u_2 + v_1 - c_{21} = 1 + 0 - 2 = -1$	
$\bar{c}_{31}$	$u_3 + v_1 - c_{31} = -1 + 0 - 2 = -3$	
$\bar{c}_{32}$	$u_3 + v_2 - c_{32} = -1 + 0 - 4 = -5$	

	$v_1=0$	$v_2=-1$	$v_3=3$
$u_1=-2$	ε [0]	[2]	5 [1]
$u_2=2$	5 [2]	5 [1]	0 [5]
$u_3=2$	[2]	[4]	5 [3]

$$\bar{c}_{ij} = U_i + V_j - c_{ij}$$

Non-basic variables $\bar{c}_{ij}$

$\bar{c}_{11}$: $U_1 + V_1 - c_{11} = -2 + 0 - 0 = -2$

$\bar{c}_{12}$: $U_1 + V_2 - c_{12} = -2 - 1 - 2 = -5$

$\bar{c}_{31}$: $U_3 + V_1 - c_{31} = 2 + 0 - 2 = 0$ 87

$\bar{c}_{32}$: $U_3 + V_2 - c_{32} = 2 - 1 - 4 = -3$

Since none of the coefficient is most +ve Hence the last table is optimal.

Total cost = $5 \times 1 + 5 \times 2 + 5 \times 1 + 5 \times 3 = 35$

A/2016. (see Q's).

Qno5(a) Solve the problem using UV-method with VAM as starting solution.

			Supply
5	1	7	10
6	4	6	80
3	2	5	15
Demand 75	20	50	

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Sol. Total supply = 105

Total Demand = 145, Difference = 40.

Step I. Starting feasible solution.

First we find starting basic feasible solution using VAM.

			supply		row penalty
5	1 10	7	10	0	5-1=4
6	4	6	80		6-4=2
3	2	5	15		3-2=1
Demand 75	20	50			
column penalty 6-5=1	2-1=1	6-5=1			

				row penalty
5	10 1	7	10 0	5-1 = 4
6	4	6	80	6-4 = 2
3	2	5	15	3-2 = 1
5	3	2	40	3-2 = 1
75	20 10	50		

column penalty 5-3=2 2-1=1 5-2=3

				row penalty
6	4	6	80	6-4=2
3	2	5	15	3-2=1
5	3	40 2	40 0	3-2=1
75	10	50 10		

column penalty 5-3=2 3-2=1 5-2=3

				row penalty
6	4	6	80	6-4=2
15 3	2	5	15 0	3-2=1
60 75	10	10		

60 (6)	(4)	(6)	80 20
60 0	10	10	

row penalty
6-4=2

column penalty: 6 4 6

(4)	10 (6)	20 10
10	10 0	

row penalty
6-4=2

0
0

column penalty: 4 6

10 (4)	10 0
10 0	

Thus starting basic feasible solution is

(5)	10 (1)	(7)
60 (6)	10 (4)	10 (6)
15 (3)	(2)	(5)
(5)	(3)	40 (2)

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Step II. Method of multipliers.

	$v_1 = 3$	$v_2 = 1$	$v_3 = 3$
$u_1 = 0$	(5)	10 (11)	(7)
$u_2 = 3$	60 (6)	10 (4)	10 (6)
$u_3 = 0$	15 (3)	(2)	(5)
$u_4 = -1$	(5)	(3)	40 (2)

Non basic variables $\qquad \bar{C}_{ij} = u_i + v_j - c_{ij}$

$\bar{C}_{11}$ $\qquad u_1 + v_1 - c_{11} = 0 + 3 - 5 = -2$

$\bar{C}_{13}$ $\qquad u_1 + v_3 - c_{13} = 0 + 3 - 7 = -4$

$\bar{C}_{32}$ $\qquad u_3 + v_2 - c_{32} = 0 + 1 - 2 = -1$

$\bar{C}_{33}$ $\qquad u_3 + v_3 - c_{33} = 0 + 3 - 5 = -2$

$\bar{C}_{41}$ $\qquad u_4 + v_1 - c_{41} = -1 + 3 - 5 = -3$

$\bar{C}_{42}$ $\qquad u_4 + v_2 - c_{42} = -1 + 1 - 3 = -3$

Since none of the coefficient is most +ve. Hence the solution.

Total cost $= 10 \times 11 + 60 \times 6 + 10 \times 4 + 10 \times 6 + 15 \times 3 + 40 \times 2$

$= 595.$

A/2015.

Qno 4 Compare the starting solutions of the following model

i) North west Corner Method.

ii) Least Cost Method.

iii) Vogel's Approximation.

10	2	20	11	15
12	7	9	20	25
4	14	16	18	10
Demand 5	15	15	15	

Sol.

i) North west corner Method

5 (10)	10 (2)	(20)	(11)	~~15~~ 10 0
(12)	5 (7)	15 (9)	5 (20)	~~25~~ 20 5 0
(4)	(14)	(16)	10 (18)	10
~~5~~ 0	~~15~~ ~~5~~ 0	~~15~~ 0	~~15~~ 10	

Thus, total cost is $\Rightarrow 5\times10 + 10\times2 + 5\times7 + 15\times9 + 5\times20 + 10\times18 = 520$

ii) Least - cost method.

10	15 (2)	20	11	15 0
12	0 (7)	15 (9)	10 (20)	25 10 0
5 (4)	14	16	5 (18)	10 5
5 0	15 0	15 0	15 10 0	

total cost $= 15\times2 + 15\times9 + 10\times20 + 5\times4 + 5\times18 = 475$

iii) Vogel's Approx method

					row penalty
10	2	20	11	15	$10-2=8$
12	7	9	20	25	$9-7=2$
5 (4)	14	16	18	10 5	$14-4=10$
5 0	15	15	15		
column penalty: $10-4=6$	$7-2=5$	$16-9=7$	$18-11=7$		

				row penalty
15 (2)	(10)	(11)	~~15~~ 0	10 − 2 = 8
(7)	(9)	(20)	25	9 − 7 = 2
(14)	(16)	(12)	5	16 − 14 = 2
~~15~~ 0	15	15		
column penalty 7 − 2 = 5	10 − 9 = 1	12 − 11 = 7		

				row penalty
0 (7)	(9)	(20)	25	9 − 7 = 2
(14)	(16)	(12)	5	16 − 14 = 2
0	15	15		
column penalty 14 − 7 = 7	16 − 9 = 7	20 − 12 = 2		

			row penalty
15 (9)	(20)	~~25~~ 10	20 − 9 = 11
(16)	(12)	5	12 − 16 = 2
~~15~~ 0	15		
column penalty 16 − 9 = 7	20 − 12 = 2		

		row penalty
10 (cost 20)	10	20
(cost 12)	5	12
~~15~~ 5		

column penalty 20 − 18 = 2

5 (cost 18)	5
5	

Thus

total cost = 5×4 + 15×2 + 0×7 + 15×9 + 10×20 + 5×18

= ~~18~~ ~~185~~ 475

Thus starting basic solution is

(10)	15 (2)	(20)	(11)
(12)	0 (7)	15 (9)	10 (20)
5 (4)	(14)	(16)	5 (18)

A/2012.

Qno 5 (b) Solve by Vogel's approx. method

7	9	9	5
3	8	18	8
5	6	5	14
7	5	3	7
9	18	7	

Sol

				row penalty
7	9	9	5	9-7 = 2
8 3	8	18	~~8~~ 0	8-3 = 5
5	6	5	14	6-5 = 1
7	5	3	7	5-3 = 2
~~9~~ 1	18	7		
column penalty 5-3=2	6-5=1	5-3=2		

column penalty

column penalty

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				row penalty
7	9	9	5	9-7 = 2
5	6	5	14	6-5 = 1
7	5	7 3	~~7~~ 0	5-3 = 2
1	18	~~7~~ 0		

column penalty: 5-7 = 2, 6-5 = 1, 5-3 = 2

			row penalty
7	9	5	9-7 = 2
5	6	14	6-5 = 1
7	0 5	0	7-5 = 2
1	18		

column penalty: 7-5 = 2, 6-5 = 1

			row penalty
7	9	5	9-7=2
5	14 6	~~14~~ 0	6-5=1
1	~~18~~ 4		

column penalty: 7-5=2, 9-6=3

			row penalty
7	4 9	~~5~~ 1	9-7=2
1	~~4~~ 0		

∞ 0

column penalty: 7, 9

1 7	1
1	

Thus the starting basic feasible solution is

1 (7)	4 (9)	(9)
8 (3)	(8)	(18)
(5)	14 (6)	(5)
(7)	0 (5)	7 (3)

A/2012

Q no 5(a) Write a note on Vogel's Approx. Method

Step I For each row (column), determine a penalty measure by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column).

Step II. Identify the row with the largest penalty, breaking ties arbitrarily. Allocate as much as possible to the variable with the least unit cost in the selected row or column. Adjust the supply and demand, and cross out the satisfy row or column. If a row and a column are satisfied simultaneously, only one of the two is crossed out, and the remaining row

(column) is assigned zero supply (demand).

Step III.

(a) If exactly one row or column with zero supply or demand remains uncrossed out, stop.

(b) If one row (column) with positive supply (demand) remain uncrossed out, determine the basic variables in the row (column)

100 by least cost method method. Stop.

(c) If all the uncrossed-out rows and columns have (remaining) zero supply and demand, determine the zero basic variables by the least cost method. Stop.

(d) Otherwise, go to step 1

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Assignment Model

A/2017.

Qno5(a). Write a note on assignment model.

Suppose we are assign m jobs to n machines. Let C_{ij} be the cost paid when a job is assigned to machine.

C_{11}	C_{12}	C_{13} . . .	C_{1n}
C_{21}	C_{22}	C_{23} . . .	C_{2n}
C_{31}	C_{32}	C_{33} . . .	C_{3n}
C_{41}	.	. . . .	.
.	.	.	.
.	.	.	.
C_{m1}	C_{m2}	C_{m3} . . .	C_{mn}

The objective is to assign the jobs to the machines at the least total cost.

The situation is known as "assignment model" Thus assignment model can be regarded as a special case of transportation model.

The job represents "sources" and the machines "destinations".

Hence the supply and demand at each source node and destination must be 1

i.e, $a_i = 1 = b_j$ $\forall i, j$

If a job can not be assign to a (source) certain machine, then the corresponding cost will be taken M(A very high cost).

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Mathematical Model.

$$x_{ij} = \begin{cases} 0, & \text{If the ith job is not assigned to the jth machine.} \\ 1, & \text{If the ith job is assigned to the jth machine.} \end{cases}$$

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Minimize. $z = \sum_{j=1}^{n}\sum_{i=1}^{n} c_{ij}\, x_{ij}$

Subject to. $\sum_{j=1}^{n} x_{ij}$, $i = 1, 2, 3, \dots, n$

$\sum_{i=1}^{n} x_{ij}$, $j = 1, 2, 3, \dots, n$, $x_{ij} = 0$ or 1

Working Rule

1- Generate zero in every row and every column by smallest cost element from the corresponding row or column and apply the assignment of jobs if it is feasible then optimal solution is

$$\sum p_i + \sum q_i$$

and if not, move to step 2

2- Draw minimum number of horizontal and vertical lines which covers all the zeros then the smallest uncrossed element

which is appearing at the intersection of two lines

3- Find the assignment of jobs if yes, then the solution is
$\sum p_i + \sum q_i + t_1$, t_1 is the smallest uncovered chosen element.

If not than continue the process.

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Q no 5(b) Find the optimal assignment by Hungarian Method.

Operators

	9	22	58	11	19	27
	43	78	72	50	63	48
Machines	41	28	91	37	45	33
	74	42	27	49	39	32
	36	11	57	22	25	18
	3	56	53	31	17	28

So

Sol.

9	22	58	11	19	27
43	78	72	50	63	48
41	28	91	37	45	33
74	42	27	49	39	32
36	11	57	22	25	18
3	56	53	31	17	28

$P_1 = 9$

$P_2 = 43$

$P_3 = 28$

$P_4 = 27$

$P_5 = 11$

$P_6 = 3$

0	13	49	2	10	18
0	35	29	7	20	5
13	0	63	9	17	5
47	15	0	22	12	5
25	0	46	11	14	7
0	53	50	28	14	25

$q_1 = 0$ $q_2 = 0$ $q_3 = 0$ $q_4 = 2$ $q_5 = 10$ $q_6 = 5$

	×	×	×			×
×	0	13	49	0	0	13
×	0	35	29	5	10	0
×	13	0	63	7	7	0
×	47	15	0	20	2	0
×	25	0	46	9	(4)	2
×	0	53	50	26	9	20

minimum uncrossed element = 4.

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	×	×	×	×	×	×
×	4	17	49	0	0	13
×	0	35	25	1	6	0
×	13	0	59	3	3	0
×	51	19	0	20	2	4
×	25	0	42	5	0	2
×	0	53	46	22	0	20

Thus minimum cost D $= \sum P_i + \sum q_i + t_1 = 9+43+28+27+11+3+2+10+5+4 = 142.$

A/2016.

Q no 5(b) Find all possible assignments using the Assignment Model

5	5	—	2
7	4	2	3
9	3	5	—
7	2	6	7

Sol

5	5	—	2	$P_1 = 2$
7	4	2	3	$P_2 = 2$
9	3	5	—	$P_3 = 3$
7	2	6	7	$P_4 = 2$

3	3	—	0
5	2	0	1
6	0	2	—
5	0	4	5
$q_1 = 3$	$q_2 = 0$	$q_3 = 0$	$q_4 = 0$

[0]

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0	3	–	0
2	2	0	1
3	0	2	–
2	0	4	5

Thus minimum cost is $= \sum p_i + \sum q_i = 2+2+3+2+3 = 12$

A/2015.

Qno 5(b). Find all possible assignments using assignment model

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3	9	2	3	7
6	1	5	6	6
9	4	7	10	3
2	5	4	2	1
9	6	2	4	6

Sol

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3	9	2	3	7	$p_1 = 2$
6	1	5	6	6	$p_2 = 1$
9	4	7	10	3	$p_3 = 3$
2	5	4	2	1	$p_4 = 1$
9	6	2	4	6	$p_5 = 2$

1	7	0	1	5
5	0	4	5	5
6	1	4	7	0
1	4	3	1	0
7	4	0	2	4

$q_1 = 1$ $q_2 = 0$ $q_3 = 0$ $q_4 = 1$ $q_5 = 0$

0	7	0	6	5
4	0	4	4	5
5	1	4	6	0
0	4	3	0	0
6	4	0	1	4

Thus the minimum cost is $= \sum p_i + \sum q_i = 2+1+3+1+2+1+1 = 11$.

Q A/2013

Qno 5(b)

Machines \ Jobs	1	2	3	4	5
I	62	78	50	101	82
II	70	85	60	75	55
III	88	96	112	85	71
IV	48	64	87	77	80
V	60	70	98	66	83

Scanned with CamScanner

62	78	50	101	82	$P_1 = 50$
70	85	60	75	55	$P_2 = 55$
88	96	118	85	71	$P_3 = 71$
48	64	87	77	80	$P_4 = 48$
60	70	98	66	83	$P_5 = 60$

12	28	0	51	32
15	30	5	20	0
17	25	47	14	0
0	16	39	29	32
0	10	38	6	23

$q_1=0$ $q_2=10$ $q_3=0$ $q_4=6$ $q_5=0$

Scanned with CamScanner

	x	x	x	x	x
x	12	18	0	45	32
x	15	20	5	14	0
x	17	15	47	8	0
x	0	6	39	23	32
x	0	0	38	0	23

Thus the minimum cost $D = \sum p_i + \sum q_i = 50+55+71+48+60+10+6 = 300$

A/2012.

Qno4b, Solve the following assignment model

	1	2	3	4
1	15	13	14	17
2	11	12	15	13
3	13	12	10	11
4	15	17	14	16

Sol.

Th

	1	2	3	4
1	15	13	14	17
2	11	12	15	13
3	13	12	10	11
4	15	17	14	16

$P_1 = 13$

$P_2 = 11$

$P_3 = 10$

$P_4 = 14$

	1	2	3	4
1	2	0	1	3
2	0	1	4	2
3	3	2	0	1
4	1	3	0	2

$q_1 = 0$ $\quad q_2 = 0$ $\quad q_3 = 0$ $\quad q_4 = 1$

		1 x	x 2	x 3	x 4
1	x	2	0	1	2
2	x	0	1	4	1
3	x	3	2	0	0
4	x	1	3	0	1

Thus the minimum cost is $= \sum p_i + \sum q_i = 13 + 11 + 10 + 14 + 1 = 49$.

A/2011.

Qno4(b) : same A/2012 Qno4(b) , A/2010 Qno5(b)

Qno5(a) : same A/2017 Qno5(a)

Qno5(b) Find the optimal assignment for the given problem.

	I	II	III	IV
A	6	4	7	6
B	7	7	1	6
C	5	9	2	8
D	5	3	5	7

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Sol

	I	II	III	IV	
A	6	4	7	6	$P_1 = 4$
B	7	7	1	6	$P_2 = 1$
C	5	9	2	8	$P_3 = 2$
D	5	3	5	7	$P_4 = 3$

Scanned with CamScanner

	I	II	III	IV
A	2	0	3	2
B	6	6	0	5
C	3	7	0	6
D	2	0	2	4

$q_1 = 2$ $q_2 = 0$ $q_3 = 0$ $q_4 = 2$

	I	II	III	IV
A	0	0	3	0
B	4	6	0	3
C	1	7	0	4
D	0	0	2	2

Thus the minimum cost is $= \sum p_i + \sum q_i = 4+1+2+3+2+2 = 14$.

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Scanned with CamScanner

Sec II. Network Models.

1- Shortest path algorithm or Dijstra's Algorithm

Let u_i be the distance from source node 1 to node i and define $d_{ij} \geq 0$ as the length of the arc (i, j).

Define the label for an immediately succeeding node j, as

$$\{u_j, i\} = \{u_i + d_{ij}, i\} \quad , \quad d_{ij} \geq 0.$$

The label for starting node is $[0, -]$ indicating that the node has no predecensor node. Label in this algorithm one of two types, temporary and permanent.

A temporary label is mentioned if a shorter route can be found; if no better route can be found, the state of the temporary label is changed to permanant.

Step 0. label the source node with permanant label.

Step 1.

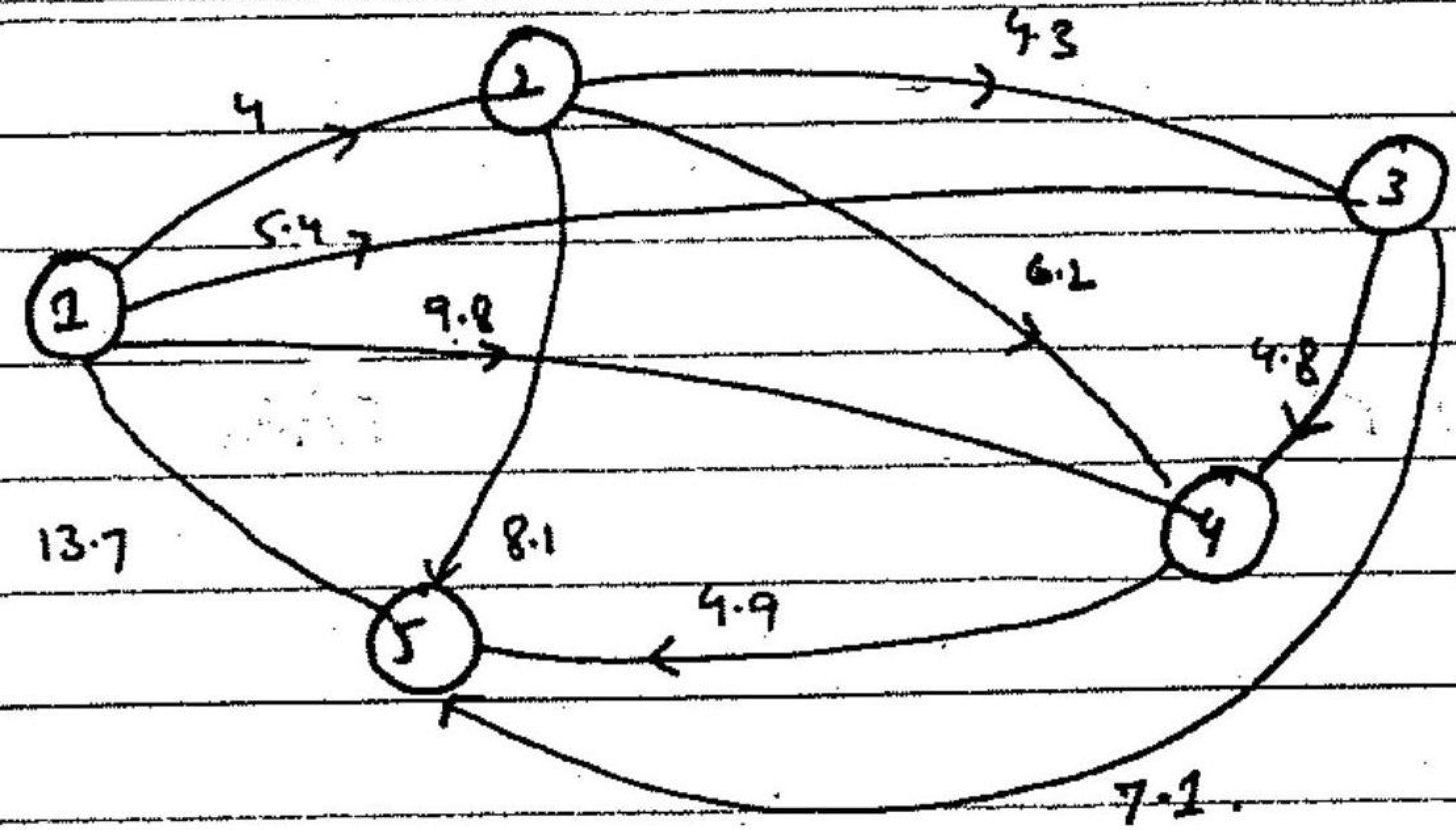
Compute the temporary labels $u_i + d_{ij}$ for each j that can be reached from node i, provided j is not permanantly labeled. If node j is already labeled with $[u_j, k]$ through another node then select

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$$\min\left\{[u_j, k],\ [u_i + d_{ij}, i]\right\}.$$

Stop otherwise repeat step 1.

Q.



Sol

Iteration # 0.

Assign the permanent label [0,-] to node 1.

Iteration # 1. Nodes 2, 3, 4 and 5 are reached from node 1.

Node	label	Status
1	[0, -]	Permanent
2	[4, 1]	Temporary
3	[5.4, 1]	Temporary
4	[9.8, 1]	Temporary
5	[13.7, 1]	Temporary

Scanned with CamScanner

Since $\min(4, 5.4, 9.8, 13.7) = 4.$

So node 2 is permanent.

Iteration # 2.

Nodes	Label	Status.
1	[0, -]	P
2	[4, 1]	P
3	[4.3, 2]	$\overline{T}$
120 4	[6.2, 2]	$\overline{T}$
5	[8.1, 2]	T

Since $\min(4.3, 6.2, 8.1) = 4.3$

So, node 3 is permanent.

Iteration #3.

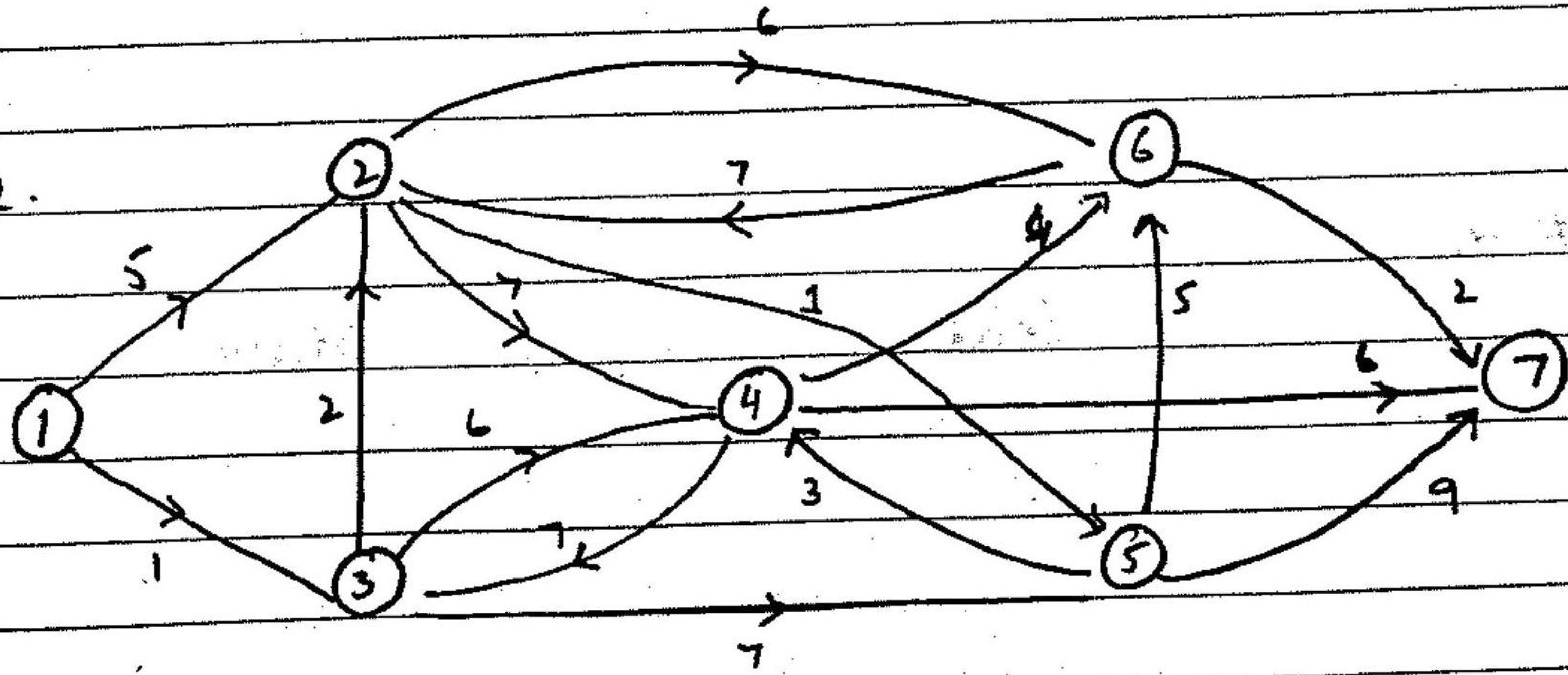
Nodes	Label	Status.
1	[0, -]	P
2	[4, 1]	P
3	[4.3, 2]	P

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	[9.8, 1]	P
4		P
5	[12.1, 1]	

Thus 1 → 2 → 5 is the shortest path and min cost is 12.1.

Example.



Sol.

Iteration #0.

Assign permanent label [0, –] to node 1.

Iteration #1.

Nodes	Label	Status.
1	[0, -]	P
2	[5, 1]	T
3	[1, 1]	T

Since min(5, 1) = 1, so node 3 is permanent.

Iteration #2.

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Nodes	Label	Status
1	[0, -]	P
2	[3, 3]	T
3	[1, 1]	P
4	[7, 3]	T
5	[8, 3]	T

Since min(3, 7, 8) = 3, so node 2 is permanent.

Iteration # 3.

Nodes	Label	Status.
1	[0, -].	P
2	[3, 3]	P
3	[1, 1]	P
4	[7, 3]	T
5	[4, 2]	T
6	[9, 2]	T

Since min (7, 4, 9) = 4 , so node 5 is permanent.

Iteration # 4.

Nodes	Label	Status.
1	[0, -]	P
2	[3, 3]	P
3	[1, 1]	P
4	[7, 3]	T
5	[4, 2]	P
6	[9, 5]	T
[illegible]	[illegible]	T

Since min(7, 9, 13) = 7, so node 4 is permanent.

Iteration #5.

Nodes	Label	Status.
1	[0, -]	P
2	[3, 3]	P
3	[1, 1]	P
4	[7, 3]	P
5	[9, 2]	P
6	[9, 5]	T
7	[13, 4]	T

Since min(9, 13) = 9, so node 6 is permanent.

Iteration #6.

Nodes	Label	Status.
1	[0, -]	P
2	[3, 3]	P
3	[1, 1]	P

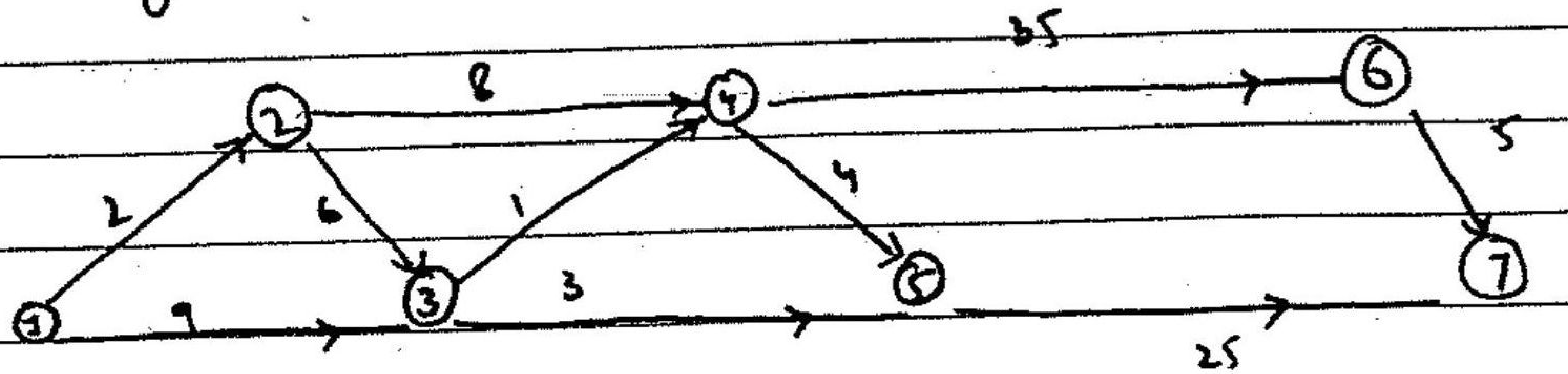
4	[7,3]	P
5	[4,2]	P
6	[9,5]	P
7	[11,6]	P

Thus the shortest path is

1 → 3 → 2 → 5 → 6 → 7

A/2015.

Qno 6(a) For the following network, the distance in miles are given b/w differant cities from city 1 to 7. Find the shortest route b/w city 1 and 7.



Sol.

Iteration #0. Assign the permanent label [0, –] to node 1.

Iteration #1.

Nodes	Label	Status.
1	[0, –]	P
2	[2, 1]	T
3	[9, 1]	T

126 Since min (2, 9) = 2, so node 2 is permanent.

Iteration #2.

Nodes	Label	Status.
1	[0, –]	P
2	[2, 1]	P
3	[8, 2]	T
4	[10, 2]	T

Since min (8, 10) = 8, so node 3 is permanent.

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Iteration # 3.

Nodes	Label	Status
1	[0, -]	P
2	[2, 1]	P
3	[8, 2]	P
4	[9, 3]	T
5	[11, 3]	T

Since min (9, 11) = 9, so node 4 is permanent

Iteration # 4.

Nodes	Label	Status
1	[0, -]	P
2	[2, 1]	P
3	[8, 2]	P
4	[9, 3]	P
5	[11, 3]	T
6	[45, 4]	T

Since min(11, 49) = 11, so node 5 is permanent

Iteration #5.

Nodes	Label	Status
1	[0, –]	P
2	[2, 1]	P
3	[8, 2]	P
4	[9, 3]	P
5	[11, 3]	P
7	[31, 5]	P

Thus shortest path b/w city 1 and 7 is

1 → 2 → 3 → 5 → 7

A

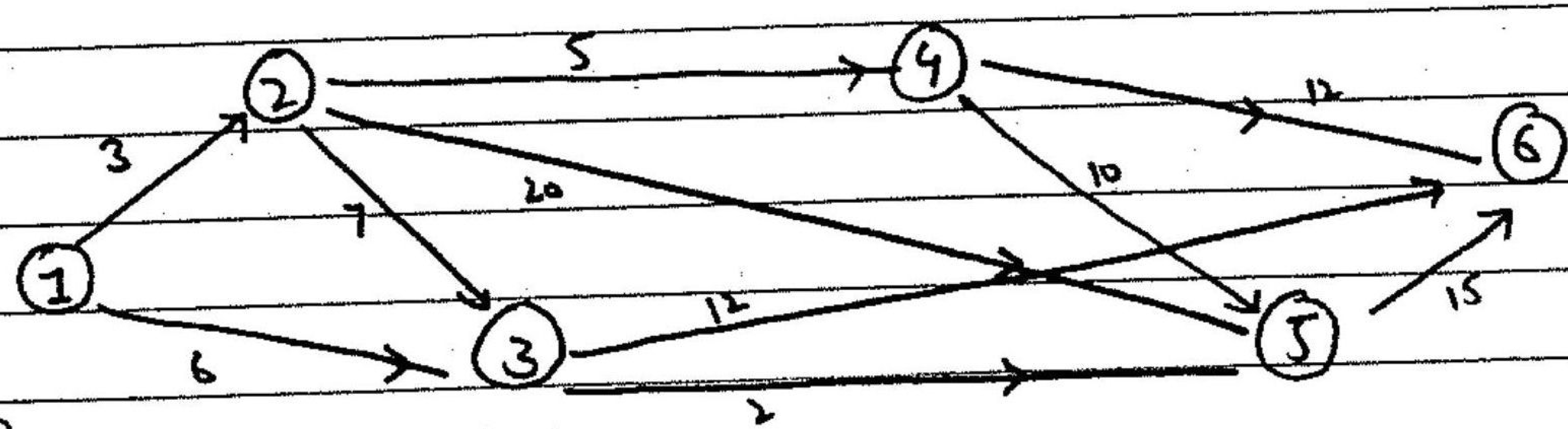
Q

So

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A/2014

Qno 9(b), Find the shortest route b/w city 1 and 6.



Sol

Iteration #0. Assign the permanent label [0, –] to node 1

Iteration #1

Nodes	Label	Status.
1	[0, –]	P
2	[3, 1]	T
3	[6, 1]	T

Since min(3, 6) = 3, so node 2 is permanent.

Iteration #2.

Nodes	Label	Status
1	[0, -]	P
2	[3, 1]	P
3	[6, 1]	T
4	[8, 2]	T
5	[23, 2]	T

∴ Since min (6, 8, 23) = 6, so node 3 is permanent

Iteration # 3

Nodes	Label	Status.
1	[0, -]	P
2	[3, 1]	P
3	[6, 1]	P
5	[8, 3]	T
6	[18, 3]	T

Since min (8, 18) = 8, so node 5 is permanent

Iteration #4:

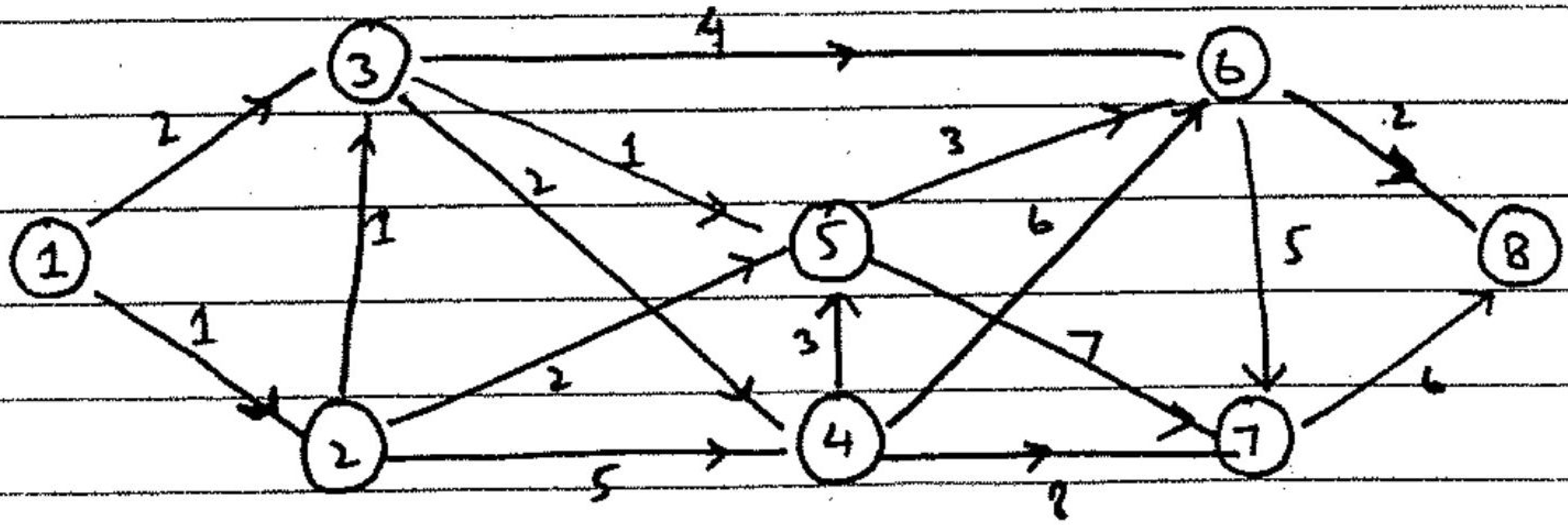
Nodes	Label	Status.
1	[0, -]	P
2	[3, 1]	P
3	[6, 1]	P
5	[8, 3]	P
6	[13, 3]	P

Thus the shortest route between city 1 and 6 is

1 ⟶ 3 ⟶ 6

A/2014.

Q no 6 Use Dijkstra's algorithm to find the shortest route b/w the following cities



a) cities 1 to 8

b) cities 2 to 8.

Sol. Iteration # 0. Assign the permanent label [0, –] to node 1.

Iteration # 1

Nodes	Label	States.
1	[0, –]	P
2	[1, 1]	T
3	[2, 1]	T

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Since min (2, 1) = 1 , so node 2 is permanent

Iteration # 2.

Nodes	Label	Status
1	[0, -]	P
2	[1, 1]	P
3	[2, 1]	T
4	[6, 2]	T
5	[3, 2]	T

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Since min (2, 6, 3) = 2 , so node 3 is permanent

Iteration # 3.

Nodes	Label	Status
1	[0, -]	P
2	[1, 1]	P
3	[2, 1]	P
4	[4, 3]	T
5	[3, 2]	T
6	[6, 3]	T

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Since min(4,3,6) = 3 so node 5 is permanent.

Iteration # 4.

Nodes	Label	Status.
1	[0, -]	P
2	[1, 1]	P
3	[2, 1]	P
4	[4, 3]	T
134 5	[3, 2]	P
6	[6, 5]	T
7	[10, 5]	T

Since min (4, 6, 10) = 4, so node 4 is permanent.

Iteration #5.

Nodes	Label	Status.
1	[0, -]	P
2	[1, 1]	P
3	[2, 1]	P

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4	[4, 3]	P
5	[3,2]	P
6	~~min [8,4]~~ [6, 5]	T
7	[10, 5]	T

Since min (6, 10) = 6, so node 6 is permanent.

Iteration # 6.

Nodes	Label	Status.
1	[0,-]	P
2	[1,1]	P
3	[2,1]	P
4	[4,3]	P
5	[3,2]=[3,3]	P
6	[6,5]	P
7	[10,5]	T
8	[8,6]	P

Since min (10, 8) = 8, so node 8 is permanent

Scanned with CamScanner

Thus the shortest route is

$1 \rightarrow 3 \rightarrow 5 \rightarrow 6 \rightarrow 8$

(b) Shortest distance b/w City 2 and 6.

Iteration # 0. Assign permanent label [0, -] to node 2.

Iteration # 1.

Nodes	Label	Status
2	[0, -]	P
3	[1, 2]	T
4	[5, 2]	T
5	[2, 2]	T

Since min (1, 5, 2) = 1 ; so node 3 is permanent.

Iteration # 2.

Nodes	Label	Status
2	[0, -]	P
3	[1, 2]	P

4	[3,3]	T
5	[2,2]	T
6	[5,3]	T

Since min (3,2,5) = 2 so node 5 is permanent

Iteration # 3.

Nodes	Label	Status.
2	[0,-]	P
3	[1,2]	P
4	[3,3]	P
5	[2,2]	P
6	[5,3]	T

Since min (3, 5) = 3, so node 4 is permanent.

Iteration # 4.

Nodes	Label	Status.
2	[0,-]	P
3	[1,2]	P

4	[3, 3]	P
5	[2, 2]	P
6	[5, 3]	P

Thus shortest path b/we cities 2 and 6 is
2 → 3 → 6.

Advanced Linear Programing.

1- Revised Simplex Method.

A/2017
Qno6(a) Write a note on Revised Simplex Method

Step 0. Construct a starting basic feasible solution, and let B and C_B be its associated Basis and objective coefficient vectors respectively.

Step 1. Compute the inverse B^{-1} of the Basis B by using appropriate inversion method

Step 2. For each non-basic vector P_j, compute 130

$$z_j - c_j = C_B B^{-1} P_j - c_j$$

- If $z_j - c_j \geq 0$ in maximization (≤ 0 in minimization) for all non basic vectors, stop; the optimal solution is $X_B = B^{-1} b$, $z = C_B X_B$
  Else, determining the entering vector P_j have the most -ve (positive) in maximization (minimization) problem among all the non-basic vectors.

Step 3. Compute $B^{-1} P_j$, If all the elements of $B^{-1} P_j$ are negative or zero, stop; the solution is unbounded. Else use the ratio test to determine the leaving-

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Step 4. From the next basis by replacing the leaving vector P_r with the entering vector P_j in the current basis B. Go to step 1 and start new iteration

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A/2015.

Qno 6(b) Use Revised Simplex Method to solve.

Maximize: $z = 4x_1 + 3x_2 + 6x_3$

Subject to $3x_1 + x_2 + 3x_3 \leq 30$

$2x_1 + 2x_2 + 3x_3 \leq 40$

$x_1, x_2, x_3 \geq 0.$

Sol. Write the LP model in standard form.

Max $z = 4x_1 + 3x_2 + 6x_3 + 0x_4 + 0x_5$

Subject to $3x_1 + x_2 + 3x_3 + x_4 = 30$

$2x_1 + 2x_2 + 3x_3 + x_5 = 40.$

$x_1, x_2, x_3, x_4, x_5 \geq 0.$

Let P_i, $i = 1, 2, \ldots 4, 5$ denotes the column vectors corresponding to the variables $x_1, x_2, \ldots x_4, x_5$ respectively

Also let 'b' denotes the R.H.S of the constraints i.e, $b = \begin{pmatrix} 30 \\ 40 \end{pmatrix}$

$P_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $P_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $P_3 = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$, $P_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $P_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

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Iteration #0. For starting solution take $x_1 = x_2 = x_3 = 0$.

Basis is

$$B = [P_4 \quad P_5] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Solution column is $= \bar{b} = B^{-1} b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 40 \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \end{bmatrix}.$

- **Z-coefficient of non-basic variables.**

$$M = C_B B^{-1} = [0 \quad 0] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [0 \quad 0].$$

- Z-coefficient of $x_1 = Z_1 - C_1 = C_B B^{-1} P_1 - C_1 = [0 \;\; 0] \begin{bmatrix} 3 \\ 2 \end{bmatrix} - 4 = -4$

- Z-coefficient of $x_2 = Z_2 - C_2 = MP_2 - C_2 = [0 \;\; 0] \begin{bmatrix} 1 \\ 2 \end{bmatrix} - 3 = -3$

- Z-coefficient of $x_3 = Z_3 - C_3 = MP_3 - C_3 = [0 \;\; 0] \begin{bmatrix} 3 \\ 3 \end{bmatrix} - 6 = -6$

The most +ve coefficient is of x_3 variable so x_3 enters the system and pivot column is

$$\bar{P}_3 = B^{-1} P_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

Basis	B^{-1}	solution column	entering variable	Pivot column	ratio
$\leftarrow x_4$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	30	x_3	3	10
x_5		40		3	$40/3$

So P_4 leaves the Basis.

Iteration #1.

$$B = [P_3 \;\; P_5] = \begin{bmatrix} 3 & 0 \\ 3 & 1 \end{bmatrix} \Rightarrow B^{-1} = \begin{bmatrix} 1/3 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\text{Solution column} = \underline{b} = B^{-1} b = \begin{bmatrix} 1/3 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 40 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

z-coefficient of non-basic variable

$$M = C_B B^{-1} = [6 \;\; 0] \begin{bmatrix} 1/3 & 0 \\ -1 & 1 \end{bmatrix} = [2 \;\; 0].$$

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z-coefficient of $x_1 = z_1 - c_1 = MP_1 - c_1 = \begin{bmatrix} 2 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} - 4 = 2$

z-coefficient of $x_2 = z_2 - c_2 = MP_2 - c_2 = \begin{bmatrix} 2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} - 3 = -1$

z-coefficient of $x_4 = z_4 - c_4 = MP_4 - c_4 = \begin{bmatrix} 2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 0 = 2$

144 Thus x_2 variable has most -ve coefficient so x_2 enters the system and the pivot column is

$$\bar{P}_2 = B^{-1}P_2 = \begin{pmatrix} 1/3 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1 \end{pmatrix}$$

Basis	B^{-1}		b	Entering variable	$\bar{P}_2$	ratio
x_3	1/3	0	10	x_2	1/3	30
← x_5	-1	1	10		1	10

So P_5 leaves the basis

Iteration #2.

$B = [P_3 \quad P_2] = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix} \Rightarrow B^{-1} = \begin{bmatrix} 2/3 & -1/3 \\ -1 & 1 \end{bmatrix}$

Solution column $= \underline{b} = B^{-1}b = \begin{bmatrix} 2/3 & -1/3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 30 \\ 40 \end{bmatrix} = \begin{bmatrix} 20/3 \\ 10 \end{bmatrix}$

Z-coefficient of non-basic variables

$M = C_B B^{-1} = [-3 \quad 6]\begin{bmatrix} 2/3 & -1/3 \\ -1 & 1 \end{bmatrix} = [-4 \quad 5]$.

Z-coefficient of $x_1 = Z_1 - C_1 = MP_1 - C_1 = [-4 \quad 5]\begin{bmatrix} 3 \\ 2 \end{bmatrix} - 4 = -12 + 10 - 4 = -6$

So P_1 enters the basis and pivot column is

$\bar{P}_1 = B^{-1}P_1 = \begin{bmatrix} 2/3 & -1/3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4/3 \\ -1 \end{bmatrix}$

Basis	B^{-1}	$\underline{b}$	entering variable	$\bar{P}_1$	ratio.
$\leftarrow x_3$	$\begin{pmatrix} 2/3 & -1/3 \\ -1 & 1 \end{pmatrix}$	$20/3$	x_1	$4/3$	5
x_2		10		-1	–

P_3 leaves the basis

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Iteration #3.

$$B = [P_1 \;\; P_2] = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} \Rightarrow B^{-1} = \begin{bmatrix} 1/2 & -1/4 \\ -1/2 & 3/4 \end{bmatrix}.$$

Solution column $= \underline{b} = B^{-1} b = \begin{bmatrix} 1/2 & -1/4 \\ -1/2 & 3/4 \end{bmatrix} \begin{bmatrix} 30 \\ 40 \end{bmatrix} = \begin{bmatrix} 15-10 \\ -15+30 \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}.$

z-coefficient of non-basic.

$$M = C_B B^{-1} = [4 \;\; 3] \begin{bmatrix} 1/2 & -1/4 \\ -1/2 & 3/4 \end{bmatrix} = [1/2 \;\; 5/4]$$

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z-coefficient of $x_3 = z_3 - c_3 = MP_3 - c_3 = [1/2 \;\; 5/4] \begin{bmatrix} 3 \\ 3 \end{bmatrix} - 6 = -3/4$

z-coefficient of $x_4 = z_4 - c_4 = MP_4 - c_4 = [1/2 \;\; 5/4] \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 0 = \frac{1}{2}.$

P_3 enters the basis and pivot column is

$$\bar{P}_3 = B^{-1} P_3 = \begin{bmatrix} 1/2 & -1/4 \\ -1/2 & 3/4 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 3/2 - 3/4 \\ -3/2 + 9/4 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 3/4 \end{bmatrix}$$

Basis	B^{-1}		Solution column	entering variable	$\bar{P}_3$	ratio
← x_1	1/2	-1/4	5	x_3	3/4	20/3
x_2	-1/2	3/4	15		3/4	45/3

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A/2013.

Q no 6(a). Use the revised Simplex method to solve the following problem.

Max: $z = 3x_1 + 2x_2 + 5x_3$

Subject to:

$$x_1 + 2x_2 + x_3 \geq 430$$

$$3x_1 + 2x_2 \leq 460$$

$$x_1 + 4x_3 \leq 420.$$

$$x_1, x_2, x_3 \geq 0.$$

Sol. Convert the LP model in standard form 147

Max: $z = 3x_1 + 2x_2 + 5x_3 - Mx_4$

$$x_1 + 2x_2 + x_3 + x_4 - x_5 = 430$$

$$3x_1 + 2x_2 + x_6 = 460$$

$$x_1 + 4x_3 + x_7 = 420$$

Iteration #0

$$B = (P_4 \quad P_6 \quad P_7) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$C_B = (-M \quad 0 \quad 0)$

z- coefficient of non-basic $= C_B B^{-1} = (-M \quad 0 \quad 0) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$= (-M \quad 0 \quad 0)$

z - coefficient of $x_1 = C_B B^{-1} P_1 - C_1 = (-M \quad 0 \quad 0) \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} - 3 = -M - 3$

z - coefficient of $x_2 = C_B B^{-1} P_2 - C_2 = (-M \quad 0 \quad 0) \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - 2 = -2M - 2$

z - coefficient of $x_3 = C_B B^{-1} P_3 - C_3 = (-M \quad 0 \quad 0) \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} - 5 = -M - 5$

z - coefficient of $x_5 = C_B B^{-1} P_5 - C_5 = (-M \quad 0 \quad 0) \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} - 0 = M$

Since x_2 has most -ve coeffient, so P_2 enters the basis and pivot column

$\bar{P}_2 = B^{-1} P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$.

Basis	B^{-1}	b	entering variable	pivot column	ratio
$\leftarrow x_4$	1 0 0	430		2	215
x_6	0 1 0	460	x_2	2	230
x_7	0 0 1	420		0	—

P_4 leaves the basis.

Iteration #1.

$$B = (P_2 \quad P_6 \quad P_7) = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow B^{-1} = \begin{pmatrix} 1/2 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{solution column} = \underline{b} = B^{-1} b = \begin{pmatrix} 1/2 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 430 \\ 460 \\ 420 \end{pmatrix}$$

$$\underline{b} = \begin{pmatrix} 215 \\ 30 \\ 420 \end{pmatrix}$$

$$z\text{- coefficients of non-basic} = C_B B^{-1} = (2 \quad 0 \quad 0) \begin{pmatrix} 1/2 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= (1 \quad 0 \quad 0)$$

$$z\text{- coefficient of } x_1 = C_B B^{-1} P_1 - C_1 = (1 \quad 0 \quad 0) \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} - 3 = 1 - 3 = -2$$

z - coefficient of $x_3 = C_B B^{-1} P_3 - C_3 = (1 \quad 0 \quad 0)\begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} - 5 = 1 - 5 = -4$

z - coefficient of $x_4 = C_B B^{-1} P_4 - C_4 = (1 \quad 0 \quad 0)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - (-M) = 1+M$

Since x_3 variable has most -ve coefficient, so P_3 enters the Basis and Pivot column is

$$\bar{P}_3 = B^{-1} P_3 = \begin{pmatrix} 1/2 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 1/2 \\ -1 \\ 4 \end{pmatrix}$$

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Basis	B^{-1}			Solution column	entering variable	$\bar{P}_3$	ratio.
x_2	1/2	0	0	215		1/2	430
x_6	-1	1	0	30	x_3	-1	—
← x_7	0	0	1	420		4	105

P_7 leaves the basis.

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Iteration #2

$$B = (P_2 \quad P_6 \quad P_3) = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} \Rightarrow B^{-1} = \begin{pmatrix} 1/2 & 0 & -1/8 \\ -1 & 1 & 1/4 \\ 0 & 0 & 1/4 \end{pmatrix}$$

$$\text{Solution column} = b = B^{-1}b = \begin{pmatrix} 1/2 & 0 & -1/8 \\ -1 & 1 & 1/4 \\ 0 & 0 & 1/4 \end{pmatrix} \begin{pmatrix} 430 \\ 460 \\ 420 \end{pmatrix} = \begin{pmatrix} 325/2 \\ 135 \\ 105 \end{pmatrix}$$

$$z\text{-coefficient of non-basic} = C_B B^{-1} = (2 \quad 0 \quad 5) \begin{pmatrix} 1/2 & 0 & -1/8 \\ -1 & 1 & 1/4 \\ 0 & 0 & 1/4 \end{pmatrix}$$

$$= (1 \quad 0 \quad 1)$$

$$z\text{-coefficient of } x_1 = C_B B^{-1} P_1 - C_1 = (1 \quad 0 \quad 1) \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} - 3 = 2 - 3 = -1$$

$$z\text{-coefficient of } x_4 = C_B B^{-1} P_4 - C_4 = (1 \quad 0 \quad 1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - (-M) = 1 + M.$$

Since x_1 coefficient is most -ve. so P_1 enters the basis, $\bar{P}_1 = B^{-1}P_1 = \begin{pmatrix} 3/8 \\ 9/4 \\ 1/4 \end{pmatrix}$

Basis	B^{-1}			Solution column	entering variable	$\bar{P}_1$	
x_2	1/2	0	-1/8	325/2		3/8	2600/3
← x_6	-1	1	1/4	135	x_1	9/4	60
x_3	0	0	1/4	105		1/4	420

P_6 leaves the basis.

Iteration #3.

$$B = \begin{pmatrix} P_2 & P_1 & P_3 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & 1 & 4 \end{pmatrix} \Rightarrow B^{-1} = \begin{pmatrix} 4/5 & -1/5 & -1/5 \\ -8/15 & 7/15 & 2/15 \\ -1/15 & -1/15 & 4/15 \end{pmatrix}$$

$$\text{Solution column} = \underline{b} = B^{-1} b = \begin{pmatrix} 4/5 & -1/5 & -1/5 \\ -8/15 & 7/15 & 2/15 \\ -1/15 & -1/15 & 4/15 \end{pmatrix} \begin{pmatrix} 430 \\ 460 \\ 420 \end{pmatrix} = \begin{pmatrix} 168 \\ 124/3 \\ 158/3 \end{pmatrix}$$

$$Z\text{-coefficient of non basic} = C_B B^{-1} = \begin{pmatrix} 2 & 3 & 5 \end{pmatrix} \begin{pmatrix} 4/5 & -1/5 & -1/5 \\ -8/15 & 7/15 & 2/15 \\ -1/15 & -1/15 & 4/15 \end{pmatrix} = \begin{pmatrix} \frac{-1}{3} & \frac{2}{3} & \frac{4}{3} \end{pmatrix}$$

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$$Z\text{-coefficient of } x_4 = C_B B^{-1} P_4 - C_4 = \begin{pmatrix} \frac{-1}{3} & \frac{2}{3} & \frac{4}{3} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - (-M) = \frac{-1}{3} + M$$

$$Z\text{-coefficient of } x_5 = C_B B^{-1} P_4 - C_4 = \begin{pmatrix} \frac{-1}{3} & \frac{2}{3} & \frac{4}{3} \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} - 0 = \frac{1}{3}.$$

Since no coefficient most +ve so the solution is optimal.

Basic	B^{-1}			Solution column
x_2	4/15	-1/5	-1/5	16 8
x_4	-8/15	7/15	2/15	124/3
x_3	-1/15	-1/15	4/15	158/3

A/2013.

Qno 6 (b) Solve the following by Revised simplex Method

Minimize $Z = 12x_1 + 20x_2$

subject to $3x_1 + 4x_2 \geq 50$

$7x_1 + 12x_2 \geq 120.$

$x_1 \geq 0, \; x_2 \geq 0.$

Solution Convert the LP model to standard form

Max $Z^* = (-Z) = -12x_1 - 20x_2 + Mx_3 + Mx_4$

subject to : $3x_1 + 4x_2 + x_3 - x_5 = 50$

$7x_1 + 12x_2 + x_4 - x_6 = 120$

~~Here~~ B

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Iteration #0.

$B = (P_3 \;\; P_4) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Solution column $= \begin{pmatrix} 50 \\ 120 \end{pmatrix}$.

z-coefficient of non-basic $= C_B B^{-1} = (-M \;\; -M)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (-M \;\; -M)$.

154 z-coefficient of x_1 $= C_B B^{-1} P_1 - C_1 = (-M \;\; -M)\begin{pmatrix} 3 \\ 7 \end{pmatrix} - (-12) = -10M + 12$.

z-coefficient of x_2 $= C_B B^{-1} P_2 - C_2 = (-M \;\; -M)\begin{pmatrix} 4 \\ 12 \end{pmatrix} - (-20) = -16M + 20$.

The most negative z-coefficient is of x_2, so P_2 enters the basis and pivot column

$\bar{P}_2 = B^{-1} P_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 12 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \end{pmatrix}$.

Basis	B^{-1}	Solution column	entering variable	$\bar{P}_2$	ratio
x_3	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	50	x_2	4	$50/4 = 25/2$
← x_4		120		12	$120/12 = 10$

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P_4 leaves the basis.

Iteration # 1.

$$B = (P_3 \;\; P_1) = \begin{pmatrix} 1 & 4 \\ 0 & 12 \end{pmatrix} \Rightarrow B^{-1} = \begin{pmatrix} 1 & -1/3 \\ 0 & 1/12 \end{pmatrix}$$

Solution column $= \underline{b} = B^{-1} b = \begin{pmatrix} 1 & -1/3 \\ 0 & 1/12 \end{pmatrix}\begin{pmatrix} 50 \\ 120 \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$

z-coefficient of non-basic $= C_B B^{-1} = (-M \;\; -20)\begin{pmatrix} 1 & -1/3 \\ 0 & 1/12 \end{pmatrix} = \left(-M \;\; \frac{M-5}{3}\right)$ 155

z-coefficient of $x_1 = C_B B^{-1} P_1 - C_1 = \left(-M \;\; \frac{M-5}{3}\right)\begin{pmatrix} 3 \\ 7 \end{pmatrix} - (-12) = \frac{-2M}{3} - \frac{35}{3} + 12 = \frac{-2M+1}{3}$

z-coefficient of $x_5 = C_B B^{-1} P_5 - C_5 = \left(-M \;\; \frac{M-5}{3}\right)\begin{pmatrix} -1 \\ 0 \end{pmatrix} - 0 = M.$

Since most -ve is x_1 z-coefficient so P_1 enters the basis and pivot column

$$\bar{P}_1 = B_*^{-1} P_1 = \begin{pmatrix} 1 & -1/3 \\ 0 & 1/12 \end{pmatrix}\begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 2/3 \\ 7/12 \end{pmatrix}$$

Basis	B^{-1}		solution column	entering variable	$\bar{P}_1$	ratio.
← x_3	1	$-1/3$	10	x_1	$2/3$	15
x_2	0	$1/12$	10		$7/12$	$120/7$

P_3 leaves the basis.

Iteration #2.

$$B = (P_1 \quad P_2) = \begin{pmatrix} 3 & 4 \\ 7 & 12 \end{pmatrix} \Rightarrow B^{-1} = \begin{pmatrix} 3/2 & -1/2 \\ -7/8 & 3/8 \end{pmatrix}.$$

$$\text{Solution column} = B^{-1}b = \begin{pmatrix} 3/2 & -1/2 \\ -7/8 & 3/8 \end{pmatrix}\begin{pmatrix} 80 \\ 120 \end{pmatrix} = \begin{pmatrix} 15 \\ 5/4 \end{pmatrix}.$$

156 $$\text{Z-coefficient of non-basic} = C_B B^{-1} = (-12 \quad -20)\begin{pmatrix} 3/2 & -1/2 \\ -7/8 & 3/8 \end{pmatrix} = \left(-\frac{1}{2} \quad -\frac{3}{2}\right).$$

Since no z-coefficient is most -ve, hence the solution is optimal.

Basis	B^{-1}		Solution column
x_1	3/2	-1/2	15
x_2	-7/8	3/8	5/4

A/2011.

Qno6 Solve the problem by Revised Simplex Method.

Max: $z = 2x_1 + x_2$

$3x_1 + 4x_2 \leq 6$

$6x_1 + x_2 \leq 3.$

$x_1 \geq 0, \ x_2 \geq 0.$

Solution. Convert the given LPP to standard form.

Max $\quad z = 2x_1 + x_2 + 0x_3 + 0x_4$

subject to $\quad 3x_1 + 4x_2 + x_3 = 6.$

$6x_1 + x_2 + x_4 = 3.$

$x_1, x_2, x_3, x_4 \geq 0.$

Let P_i's be the column vector corresponding to x_i's

$P_1 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \ P_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \ P_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ P_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$

and $b = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ (R.H.S of the constraints).

For starting solution take $x_1 = x_2 = 0$

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Iteration # 0.

$B = (P_3 \quad P_4) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

solution column $= \underline{b} = B^{-1}b = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$.

Z-coefficients of non-basic $= C_B B^{-1} = (0 \quad 0)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (0 \quad 0)$.

158 Z- coefficient of $x_1 = C_B B^{-1} P_1 - C_1 = (0 \quad 0)\begin{pmatrix} 3 \\ 6 \end{pmatrix} - 2 = -2$.

Z- co-efficient of $x_2 = C_B B^{-1} P_2 - C_2 = (0 \quad 0)\begin{pmatrix} 4 \\ 1 \end{pmatrix} - 1 = -1$

Since Z- coefficient of x_1 is most -ve so P_1 enters the basis and pivot column is

$\bar{P}_1 = B^{-1} P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

Basis	B^{-1}	solution column	entering variable	$\bar{P}_1$	ratio.
x_3	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	6	x_1	3	$6/3 = 2$
$\leftarrow x_4$		3		6	$3/6 = 1/2$

P_4 leaves the Basis

Iteration #1.

$$B = (P_3 \quad P_1) = \begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix} \Rightarrow B^{-1} = \begin{pmatrix} 1 & -1/2 \\ 0 & 1/6 \end{pmatrix}$$

Solution column $= \bar{b} = B^{-1}b = \begin{pmatrix} 1 & -1/2 \\ 0 & 1/6 \end{pmatrix}\begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 9/2 \\ 1/2 \end{pmatrix}$.

Optimality Computation

1- coefficient of nonbasic $= C_B B^{-1} = (0 \; 2)\begin{pmatrix} 1 & -1/2 \\ 0 & 1/6 \end{pmatrix} = (0 \quad 1/3)$

2- coefficient of $x_2 = C_B B^{-1} P_2 - C_2 = (0 \; \frac{1}{3})\begin{pmatrix} 4 \\ 1 \end{pmatrix} - 1 = \frac{1}{3} - 1 = \frac{-2}{3}$

So P_2 enters the basis (optimality computations) and pivot column is

$\bar{P}_{x_2} = B^{-1}P_2 = \begin{pmatrix} 1 & -1/2 \\ 0 & 1/6 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 7/2 \\ 1/6 \end{pmatrix}$.

Feasibility Computations

Basis	B^{-1}	solution column	entering variable	$\bar{P}_2$	ratio
x_3	$\begin{pmatrix} 1 & -1/2 \\ 0 & 1/6 \end{pmatrix}$	$9/2$	x_2	$7/2$	$9/7$
x_1		$1/2$		$1/6$	3

So P_3 leaves the basis (by feasibility condition)

Iteration #2.

$$B = (P_2 \;\; P_1) = \begin{pmatrix} 4 & 3 \\ 1 & 6 \end{pmatrix} \Rightarrow B^{-1} = \begin{pmatrix} 2/7 & -1/7 \\ -1/21 & 4/21 \end{pmatrix}.$$

Solution column $= \underline{b} = B^{-1} b = \begin{pmatrix} 2/7 & -1/7 \\ -1/21 & 4/21 \end{pmatrix} \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{9}{7} \\ 2/7 \end{pmatrix}.$

Z-coefficient of nonbasic $= C_B B^{-1} = (1 \;\; 2) \begin{pmatrix} 2/7 & -1/7 \\ -1/21 & 4/21 \end{pmatrix} = \left(\frac{4}{21} \;\; \frac{5}{21} \right)$

160 Now, further no z-coefficient is most -ve. Hence the solution is optimal

Basis	B^{-1}		solution column.
x_2	2/7	-1/7	9/7
x_1	-1/21	4/21	2/7

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Revision.

Revised Simplex Algorithm.

$\text{Max } z = 6x_1 - 2x_2 + 3x_3$

subject to: $2x_1 - x_2 + 2x_3 \leq 2$

$x_1 + 4x_3 \leq 4$

$x_1, x_2, x_3 \geq 0.$

Sol

The given LP model is standard form

$\text{Max } z = 6x_1 - 2x_2 + 3x_3 + 0x_4 + 0x_5$

subject to $2x_1 - x_2 + 2x_3 + x_4 = 2.$

$x_1 + 4x_3 + x_5 = 4.$

$x_i \geq 0 \quad i = 1, 2, \ldots 5.$

Let P_i, $i = 1, 2, 3, 4, 5$ denotes the column vector corresponding to the variables x_i, $i = 1, 2, 3, 4, 5$, respectively.

Also let b denotes the R.H.S of the constraints i.e., $b = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

$P_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $P_2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $P_3 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $P_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $P_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

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Iteration # 0

For starting solution

$x_1 = x_2 = x_3 = 0$.

Basis is

$$B = [P_4 \quad P_5] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Solution column $= \underline{b} = B^{-1} b$.

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 0 \\ 0 + 4 \end{bmatrix}$$

$$\underline{b} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

- z-coefficients of non basic variables

$$M = C_B B^{-1} = [0 \quad 0] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [0 \quad 0]$$

$(z_j - c_j)$ most -ve

Z-coefficient of $x_1 = z_1 - c_1 = MP_1 - c_1 = \begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} - 6 = -6$

" " " $x_2 = z_2 - c_2 = MP_2 - c_2 = \begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \end{bmatrix} - (-2) = 2$

" " " $x_3 = z_3 - c_3 = MP_3 - c_3 = \begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix} - 3 = -3$

" " " $x_4 = z_4 - c_4 = MP_4 - c_4 = \begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 0 = 0$

" " " $x_5 = z_5 - c_5 = MP_5 - c_5 = \begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 0 = 0$

The most -ve is x_1. So x_1 enters. Thus P_1 enters the Basis and Pivot column is

$\bar{P}_1 = B^{-1}P_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$

Basis	B^{-1}	Solution column	entering variable	Pivot column	ratio
$\leftarrow x_4$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	2	x_1	2	$2/2 = 1$
x_5		4		1	$4/1 = 4$

So P_4 leaves the Basis.

Iteration # 1

The new basis B

$$B = [P_1 \; P_5] = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}.$$

$$B^{-1} = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix}.$$

$$\text{Solution column} = \underline{b} = B^{-1} b = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+0 \\ -1+4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

z-coefficient of non-basic variable

$$M = C_B B^{-1} = [6 \;\; 0] \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix} = [3 \;\; 0]$$

Now

$$z\text{-coefficient of } x_2 = z_2 - c_2 = MP_2 - C_2 = [3 \;\; 0] \begin{bmatrix} -1 \\ 0 \end{bmatrix} - (-2)$$

$$= -3 + 2 = -1$$

z-coefficient of $x_3 = z_3 - c_3 = MP_3 - c_3 = [3 \quad 0]\begin{bmatrix}2\\4\end{bmatrix} - 3 = 6 - 3 = 3$

" " " $x_4 = z_4 - c_4 = MP_4 - c_4 = [3 \quad 0]\begin{bmatrix}1\\0\end{bmatrix} - 0 = 3$

" " " $x_5 = z_5 - c_5 = MP_5 - c_5 = [3 \quad 0]\begin{bmatrix}0\\1\end{bmatrix} - 0 = 0$.

Since most -ve z-coefficient is x_1, so P_2 enters the Basis and Pivot column is

$$\bar{P}_2 = B^{-1}P_2 = \begin{pmatrix}1/2 & 0\\ -1/2 & 1\end{pmatrix}\begin{bmatrix}-1\\0\end{bmatrix} = \begin{bmatrix}-1/2\\1/2\end{bmatrix}$$

Basic	B^{-1}	Solution column	entering variable	pivot column	ratio
x_1	$\begin{pmatrix}1/2 & 0\end{pmatrix}$	1	x_2	$-1/2$	$1 \div (-1/2) = -2$
$\leftarrow x_5$	$\begin{pmatrix}-1/2 & 1\end{pmatrix}$	3		$1/2$	$3/\tfrac{1}{2} = 6$

Iteration # 2

The new basis is.

$$B = [P_1 \quad P_2] = \begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}.$$

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$B^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

Solution columns is $= \underline{b} = B^{-1} b$.

$= \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 + \cancel{2} \\ -\cancel{1} + 6 \end{bmatrix} = \cancel{\begin{bmatrix} 2 \\ 3 \end{bmatrix}}$

$\underline{b} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

z-coefficient of non-basic $= M = C_B B^{-1} = \begin{bmatrix} 6 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

166 ~~$M = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$~~ $M = \begin{bmatrix} 2 & 2 \end{bmatrix}$.

z-coefficient of $x_3 = z_3 - c_3 = MP_3 - c_3 = \begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} - 3$

$= 12 - 3 = 9$

Since no z-coefficient is +ve so the solution is optimal.

A/2016 Q no 6(b)

Solve by Revised Simplex Method.

Max: $z = x_1 + x_2$

Subject to: $3x_1 + 2x_2 \leq 6$

$x_1 + 4x_2 \leq 4. \quad x_1, x_2 \geq 0.$

Sol. The given LP model in standard form is

Max: $z = x_1 + x_2 + 0x_3 + 0x_4$.

Subject to: $3x_1 + 2x_2 + x_3 = 6.$

$x_1 + 4x_2 + x_4 = 4. \quad , x_1, x_2, x_3, x_4 \geq 0.$

Let P_1, P_2, P_3, P_4 be the column vectors corresponding to x_1, x_2, x_3, x_4 respectively. Let $b = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$ be the R.HS of the constraints.

$P_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, P_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, P_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$

Iteration # 0 For starting solution let $x_1 = x_2 = 0$.

The basis is

$$B = [P_3 \quad P_4] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

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Solution column $\bar{b} = B^{-1}b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

z-coefficient of non-basic variables $= M = $ ~~[0 0]~~ $C_B B^{-1} = [0 \quad 0]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [0 \quad 0]$.

z-coefficient of $x_1 = z_1 - c_1 = MP_1 - c_1 = [0 \; 0]\begin{bmatrix} 3 \\ 1 \end{bmatrix} - 1 = -1$

z-coefficient of $x_2 = z_2 - c_2 = MP_2 - c_2 = [0 \; 0]\begin{bmatrix} 2 \\ 4 \end{bmatrix} - 1 = -1$

Arbitrarily choose x_1 as entering variable.

Basic	B^{-1}	solution column	entering variable	pivot column	ratio
← x_3	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	6		3	$6/3 = 2$
x_4		4		1	$4/1 = 4$

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P_1 enter the basis and pivot column is

$\bar{P}_1 = B^{-1}P_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

Iteration #2

The new basis is

$B = [P_1 \quad P_4] = \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} 1/3 & 0 \\ -1/3 & 1 \end{bmatrix}$

~~Solution colm~~

Solution column $= \bar{b} = B^{-1}b = \begin{bmatrix} 1/3 & 0 \\ -1/3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -2+4 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$.

z-coefficient of non-basic $= M = C_B B^{-1} = [1\ 0]\begin{bmatrix} 1/3 & 0 \\ -1/3 & 1 \end{bmatrix} = [1/3\ \ 0]$

z-coefficient of x_2 $= z_2 - c_2 = MP_2 - c_2 = [\tfrac{1}{3}\ \ 0]\begin{bmatrix} 2 \\ 4 \end{bmatrix} - 1 = \frac{2}{3} - 1 = \frac{-1}{3}$

z- " " " x_3 $= z_3 - c_3 = MP_3 - c_3 = [\tfrac{1}{3}\ \ 0]\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 0 = \frac{1}{3}$

~~" " " $x_4 = z_4 - c_4 = MP_4 -$~~

Since z-coefficient of x_2 is -ve, so P_2 enters the Basis and pivot column is

$\bar{P}_2 = B^{-1}P_2 = \begin{bmatrix} 1/3 & 0 \\ -1/3 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 2/3 + 0 \\ -2/3 + 4 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 10/3 \end{bmatrix}$

Basic	B^{-1}	Solution column	entering variable	pivot column	ratio
x_1	$\begin{pmatrix} 1/3 & 0 \\ -1/3 & 1 \end{pmatrix}$	2	x_2	$2/3$	$\frac{2}{2/3} = 3$
$\leftarrow x_4$		2		$10/3$	$\frac{2}{10/3} = 3/5$

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So P_4 leaves the Basis

Iteration #2

The new basis B

$$B = [P_1 \ P_2] = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}.$$

$$B^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ -1/10 & 3/10 \end{bmatrix}.$$

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Solution column $b = B^{-1}b = \begin{bmatrix} 2/5 & -1/5 \\ -1/10 & 3/10 \end{bmatrix}\begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 8/5 \\ 3/5 \end{bmatrix}$

Z-coefficient of non-basic $= M = C_B B^{-1} = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 2/5 & -1/5 \\ -1/10 & 3/10 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} - \frac{1}{10} & -\frac{1}{5} + \frac{3}{10} \end{bmatrix}$

$$= M = \begin{bmatrix} \frac{3}{10} & \frac{1}{10} \end{bmatrix}.$$

Z-coefficient of x_3 $= z_3 - c_3 = MP_3 - c_3 = \begin{bmatrix} \frac{3}{10} & \frac{1}{10} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 0 = \frac{3}{10}$

" " " x_4 $= z_4 - c_4 = MP_4 - c_4 = \begin{bmatrix} \frac{3}{10} & \frac{1}{10} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} - 0 = \frac{1}{10}$

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Since no z-coefficient of x_i's are -ve, so the solution is optimal

A/2017 Qno 6(b)

Exp Minimize

$z = x_1 + 2x_2$.

Subject to

$2x_1 + 5x_2 \geq 6$

$x_1 + x_2 \geq 2$, $x_1, x_2 \geq 0$.

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$\frac{1+3}{10}$ Convert the given LPP into maximization problem and introduce x_3, x_4 as slack variables and x_5, x_6 as artificial variables. The problem in standard, as follow.

Max $z^* = (-z) = -x_1 - 2x_2 + 0x_3 + 0x_4 - Mx_5 - Mx_6$.

subject to

$2x_1 + 5x_2 - x_3 + x_5 = 6$

$x_1 + x_2 - x_4 + x_6 = 2$

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Using the matrix notation of given LPR

$X_B = \begin{pmatrix} x_5 \\ x_6 \end{pmatrix}$

Basis is $B = (P_5 \ P_6)$.

$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\therefore B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

$C = (-1 \quad -2 \quad 0 \quad 0 \quad -M \quad -M)$

$C_B = (-M \quad -M)$

z-coefficient of non-basic ~~var~~ $= M = C_B B^{-1} = (-M \quad -M)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (-M \quad -M)$

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z - coefficient of $x_1 = z_1 - c_1 = MP_1 - c_1 = (-M \ -M)\begin{pmatrix} 2 \\ 1 \end{pmatrix} - (-1) = -3M+1$

" " " $x_2 = z_2 - c_2 = MP_2 - c_2 = (-M \ -M)\begin{pmatrix} 5 \\ 1 \end{pmatrix} - (-2) = -6M+2$.

x_2 is entering variable, P_2 enters the basis and pivot column is $B^{-1}P_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

Basis	B^{-1}	Solution column	Entering variable	pivot column	ratio
$\leftarrow x_5$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	6	x_2	5	$6/5$
x_6		2		1	$2/1$

R_5 leaves the system.

Iteration # 1

The new basis is

$B = [P_3 \quad P_6] = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$, solution column $= b = B^{-1} b$

$= \begin{pmatrix} 1/5 & 0 \\ -1/5 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 6/5 \\ 4/5 \end{pmatrix}$

$B^{-1} = \begin{bmatrix} 1/5 & 0 \\ -1/5 & 1 \end{bmatrix}$

Z-coefficient of non-basic $= M = C_B B^{-1} = (-2 \quad -M) \begin{pmatrix} 1/5 & 0 \\ -1/5 & 1 \end{pmatrix} = \left(\frac{-2}{5} + \frac{M}{5} \quad -M \right)$

$M = \left(\frac{-2+M}{5} \quad -M \right)$

Z-coefficient of x_1 $= z_1 - c_1 = MP_1 - c_1 = \left(\frac{-2+M}{5} \quad -M \right) \begin{pmatrix} 2 \\ 1 \end{pmatrix} - (-1) = \frac{-4+2M}{5} - M + 1$

$= \frac{-3M+1}{5}$

Z-coefficient of x_3 $= z_3 - c_3 = MP_3 - c_3 = \left(\frac{-2+M}{5} \quad -M \right) \begin{pmatrix} -1 \\ 0 \end{pmatrix} - 0 = \frac{2-M}{5}$

Z- coefficient of x_4 $= z_4 - c_4 = MP_4 - c_4 = \left(\frac{-2+M}{5} \quad -M \right) \begin{pmatrix} 0 \\ -1 \end{pmatrix} - 0 = M$

Since z-coefficient of x_1 is -ve so the pivot column is

$$\bar{P} = B^{-1}P_1 = \begin{pmatrix} 1/5 & 0 \\ -1/5 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2/5 \\ -2/5+1 \end{pmatrix} = \begin{pmatrix} 2/5 \\ 3/5 \end{pmatrix}.$$

Basis	B^{-1}	solution column b	entering variable	pivot column	ratio	z-coe
x_1	$\begin{pmatrix} 1/5 & 0 \\ -1/5 & 1 \end{pmatrix}$	6/5	x_1	2/5	$\frac{6}{5}\times\frac{5}{2} = 3$	z
$\leftarrow x_6$		4/5		3/5	$\frac{4}{5}\times\frac{5}{3} = \frac{4}{3}$	

x_6 leaves the system

Iteration # 2

The new basis is $B = (P_2 \; P_1) = \begin{pmatrix} 5 & 2 \\ 1 & 1 \end{pmatrix}$

$$B^{-1} = \begin{pmatrix} 1/3 & -2/3 \\ -1/3 & 5/3 \end{pmatrix}.$$

Solution column is $b = B^{-1}b = \begin{pmatrix} 1/3 & -2/3 \\ -1/3 & 5/3 \end{pmatrix}\begin{pmatrix} 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 2-4/3 \\ -2+\frac{10}{3} \end{pmatrix} = \begin{pmatrix} 2/3 \\ 4/3 \end{pmatrix}$

z-coefficient of non-basic $= M = C_B B^{-1} = (-1 \quad -2)\begin{pmatrix} 1/3 & -2/3 \\ -1/3 & 5/3 \end{pmatrix} = \left(\frac{-1}{3}-\frac{2}{3} \quad \frac{2}{3}-\frac{10}{3}\right)$

$M = \begin{pmatrix} -1 & \frac{-8}{3} \end{pmatrix}$

z-coefficient of x_3 = $z_3 - c_3 = MP_3 - C_3 = \begin{pmatrix} -1 & \frac{-8}{3} \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} - 0 = 1$

z " " " x_4 = $z_4 - c_4 = MP_4 - c_4 = \begin{pmatrix} -1 & \frac{-8}{3} \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} - 0 = \frac{8}{3}$

" " " x_5 = $z_5 - c_5 = MP_5 - c_5 = \begin{pmatrix} -1 & \frac{-8}{3} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} - (-M) = M-1$

" " " x_6 = $z_6 - c_6 = MP_6 - c_6 = \begin{pmatrix} -1 & \frac{-8}{3} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} - (-M) = M - \frac{8}{3}$

Since no z-coefficient is most -ve, so the solution is optimal.

Basic	B^{-1}		Solution column	~~last Pivot column~~
x_2	$1/3$	$-2/3$	$2/3$	
x_1	$-1/3$	$5/3$	$4/3$	

$\frac{-10}{3}$)

$\frac{4}{3} + \frac{4}{3} = \frac{2}{3}$

Min $z = x_1 + x_2$.

Subject to.

$x_1 + 2x_2 \geq 7$

$4x_1 + x_2 \geq 6$.

$x_1, x_2 \geq 0$.

Sol. Convert the given LP into maximization LPP.

Then introduce x_3 and x_4 as slack variables and x_5 and x_6 as artificial variables

$$\text{Max } z^* = (-z) = -x_1 - x_2 + 0x_3 + 0x_4 - Mx_5 - Mx_6$$

subject to

$$x_1 + 2x_2 - x_3 + x_5 = 7$$

$$4x_1 + x_2 - x_4 + x_6 = 6$$

$$B = (P_5 \quad P_6) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$C = (-1 \quad -2 \quad 0 \quad 0 \quad -M \quad -M)$$

$$C_0 = (-M \quad -M)$$

Iteration # 0

z-coefficient of Nonbasic $= C_B B^{-1} = (-M \quad -M)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (-M \quad -M)$

z-coefficient of $x_1 = Z_1 - C_1 = C_B B^{-1} P_1 - C_1 = (-M \quad -M)\begin{pmatrix} 4 \\ 1 \end{pmatrix} - (-1) = -4M - M + 1$
$= -5M + 1$

z " " $x_2 = Z_2 - C_2 = C_B B^{-1} P_2 - C_2 = (-M \quad -M)\begin{pmatrix} 2 \\ 1 \end{pmatrix} - (-1) = -3M + 1$

" " " $x_3 = Z_3 - C_3 = C_B B^{-1} P_3 - C_3$
$= (-M \quad -M)\begin{pmatrix} -1 \\ 0 \end{pmatrix} - 0 = M$

" " " $x_4 = Z_4 - C_4 = C_B B^{-1} P_4 - C_4$
$= (-M \quad -M)\begin{pmatrix} 0 \\ -1 \end{pmatrix} - 0 = M$ 177

Since z-coefficient of x_1 is most -ve, P_1 enters the basis and pivot column is

$\bar{P}_1 = B^{-1} P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

Basis	B^{-1}	solution column(b)	entering variable	Pivot column	ratio
$\leftarrow x_5$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	7	x_1	4	$7/4$
x_6		6		1	$6/1$

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x_5 leaves the system.

Iteration # 1.

The New basis is

$$B = (P_1 \quad P_6) = \begin{pmatrix} 4 & 0 \\ 4 & 1 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 1/4 & 0 \\ -1/4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -4 & 1 \end{pmatrix}. \qquad \begin{pmatrix} 7 \\ -28+6 \end{pmatrix} = \begin{pmatrix} 7 \\ 22 \end{pmatrix}$$

Solution column $\underline{b} = B^{-1} b = \begin{pmatrix} 1/4 \; 1 & 0 \; 0 \\ -1/4 \; -4 & 1 \; 1 \end{pmatrix} \begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} 7/4 \\ -7/4 + 6 \end{pmatrix} = \begin{pmatrix} 7/4 \\ 17/4 \end{pmatrix}$

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z-coefficient of non-basic : $C_B B^{-1} = (-1 \quad -M) \begin{pmatrix} 1/4 \; 1 & 0 \; 0 \\ -1/4 \; -4 & 1 \; 1 \end{pmatrix} = \left(\frac{-1}{4} + \frac{M}{4} \quad -M \right)$ $(-1+4M \quad -M)$

$C_B B^{-1} = \left(\frac{M-1}{4} \quad -M \right)$ (corrections: $4M-1$, $\frac{-M}{2}$, $-M^2$)

z-coefficient of x_2 = $z_2 - c_2 = C_B B^{-1} P_2 - c_2 = \left(\frac{M-1}{4} \quad -M \right) \begin{pmatrix} 2 \\ 1 \end{pmatrix} - (-1) = \frac{M-1-M+1}{2}$ (corrections: $4M-1$, $-M$)

$= \frac{-M+1}{2}$ $\quad 8M - 2 - M + 1 = 7M - 1.$

z-coefficient of $x_3 = z_3 - c_3 = C_B B^{-1} P_3 - c_3 = \left(\frac{M+1}{4} \quad -M \right) \begin{pmatrix} -1 \\ 0 \end{pmatrix} - 0 = \frac{-M+1}{4}$

$(4M-1 \quad -M) \begin{pmatrix} -1 \\ 0 \end{pmatrix} = 1 - 4M$

z-coefficient of x_4 $= z_4 - c_4 = C_B B^{-1} P_4 - C_4 = (4M-1 \quad -M)\begin{pmatrix}0\\-1\end{pmatrix} - 0 = M$

z-coefficient of x_5 $= z_5 - c_5 = C_B B^{-1} P_5 - C_5 = (4M-1 \quad -M)\begin{pmatrix}1\\0\end{pmatrix} = 4M-1$

~~z-coefficient of~~

Since the most -ve z-coefficient is x_3, P_3 enters the basis and pivot column is

$$\underline{P_3} = B^{-1}P_3 = \begin{pmatrix}1 & 0\\-4 & 1\end{pmatrix}\begin{pmatrix}-1\\0\end{pmatrix} = \begin{pmatrix}-1\\4\end{pmatrix}$$

Basis	B^{-1}	Solution column	entering variable	pivot column	ratio
x_1	$\begin{pmatrix}1 & 0\\-4 & 1\end{pmatrix}$	7	x_3	-1	—
$\leftarrow x_6$		22		4	$\frac{22}{4} = \frac{11}{2}$

x_6 leaves the system

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Iteration # 2.

The new basis is

$$B = (P_1 \quad P_3) = \begin{pmatrix} 1 & -1 \\ 4 & 0 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 0 & 1/4 \\ -4 & 1/4 \end{pmatrix}$$

~~Solution column~~ $= C_B B^{-1} = (-1 \quad 0)\begin{pmatrix} 0 & 1/4 \\ -1 & 1/4 \end{pmatrix} = \begin{pmatrix} 0 \\ -1/4 + 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1/4 \end{pmatrix}$ $\begin{pmatrix} 0 & \frac{-1}{4} \end{pmatrix}$

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Z-coefficient of non-basic =

Solution column = $b = B^{-1}b = \begin{pmatrix} 0 & 1/4 \\ -1 & 1/4 \end{pmatrix}\begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 + 3/2 \\ -7 + 3/2 \end{pmatrix} = \begin{pmatrix} 3/2 \\ -11/2 \end{pmatrix}$

Z-coefficient of $x_2 = \overset{z_2 - c_2}{b} = C_B B^{-1} P_2 - c_2 =$ ~~$\begin{pmatrix} 3/2 \\ -11/2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} - (-1)$~~ $= \begin{pmatrix} 0 & \frac{-1}{4} \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} - (-1)$

$$= \frac{-1}{4} + 1 = \frac{3}{4}.$$

z-coefficient of $x_4 = Z_4 - C_4 = C_B B^{-1} P_4 - C_4 = \left(0 \;\; -\frac{1}{4}\right)\begin{pmatrix}0\\-1\end{pmatrix} - 0 = \frac{1}{4}$

z-coefficient of $x_5 = Z_5 - C_5 = C_B B^{-1} P_5 - C_5 = \left(0 \;\; -\frac{1}{4}\right)\begin{pmatrix}1\\0\end{pmatrix} = (-M) = M$

" " " $x_6 = Z_6 - C_6 = C_B B^{-1} P_6 - C_6 = \left(0 \;\; -\frac{1}{4}\right)\begin{pmatrix}0\\1\end{pmatrix} - (-M) = -\frac{1}{4} + M$

Since no z-coefficient is -ve. Hence the solution is optimal

Max. $z = x_1 + x_2$

$2x_1 + 5x_2 \le 6.$

$x_1 + x_2 \ge 2 \qquad x_1, x_2 \ge 0.$

Sol. Convert the given LP model into standard form by introducing x_3, x_4 as slack and x_5 as artificial variable.

Max. $z = x_1 + x_2 + 0x_3 + 0x_4 - Mx_5$

Subject to $2x_1 + 5x_2 + x_3 = 0$

$x_1 + x_2 - x_4 + x_5 = 0$

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Bounded Variable Algorithm

A1 2016.

Q no 7 Solve the following LPP using Bounded variable Algorithm

Max: $z = 3x_1 + 5x_2 + 2x_3$

Subject to $x_1 + x_2 + 2x_3 \leq 14$

$2x_1 + 4x_2 + 3x_3 \leq 43$

$0 \leq x_1 \leq 4$, $7 \leq x_2 \leq 10$, $0 \leq x_3 \leq 3$.

Sol Since x_2 has non-zero lower bound so first put

$x_2^* = x_2' + 7$ Then $0 \leq x_2' \leq 3$. 183

So the given LPP in standard form is

Max $z = 3x_1 + 5x_2' + 2x_3 + 35$

subject to $x_1 + x_2' + 2x_3 + x_4 = 7$

$2x_1 + 4x_2' + 3x_3 + x_5 = 15$

$0 \leq x_1 \leq 4$, $0 \leq x_2' \leq 3$, $0 \leq x_3 \leq 3$, $0 \leq x_4, x_5 \leq \infty$.

Taking x_4, x_5 as basic variables

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Basic	x_1	x_2'	x_3	x_4	x_5	sol
z	-3	-5	-2	0	0	35
x_4	1	1	2	1	0	7
x_5	2	4	3	0	1	15

Optimality condition $\Rightarrow$ x_2' is entering. Here $r = 2$, $u_2^0 = 3$.

$\theta_1 = \min\left\{\frac{7}{1}, \frac{15}{4}\right\} = \frac{15}{4}$, $\theta_2 = \infty$

184 $\theta = \min(\theta_1, \theta_2, u_2) = \min\left(\frac{15}{4}, \infty, 3\right) = 3.$

$\Rightarrow \theta = u_2.$

Replace $x_2' = 3 - x_2''$

Basic	x_1	x_2''	x_3	x_4	x_5	sol
z	-3	5	-2	0	0	50
x_4	1	-1	2	1	0	4
x_5	2	-4	3	0	1	3

for +ve

for -ve

Optimality condition $\Rightarrow$ x_1 is entering

$\theta_1 = \min\left(\frac{4}{1}, \frac{3}{2}\right) = \frac{3}{2}$, $\theta_2 = \infty$, $u_1 = 4$.

$\theta = \min(\theta_1, \theta_2, u_1) = \left(\frac{3}{2}, \infty, 4\right) = \frac{3}{2}$.

$\Rightarrow \theta = \theta_1 \Rightarrow x_5$ is leaving variable corresponding to $\frac{3}{2}$.

Basic	x_1	x_2''	x_3	x_4	x_5	sol.
z	0	-1	$5/2$	0	3	$109/2$
x_4	0	1	$1/2$	1	-1	$5/2$
x_1	1	-2	$3/2$	0	1	$3/2$

$R_2 - R_3$, $R_1 + 3R_3$

Optimality condition $\Rightarrow x_2''$ is entering ; $u_2 = 3$

for +ve entries $\rightarrow \theta_1 = \min\left\{\frac{5/2}{1}, -\right\} = \frac{5}{2}$, corresponding to basic x_4

upper bound of variable & Row corresponding

for -ve entries $\rightarrow \theta_2 = \min\left\{\frac{4 - 3/2}{-(-2)}\right\} = \frac{5}{4}$.

upper bound of entering variable.

$\theta = \min(\theta_1, \theta_2, u_2) = \min\left(\frac{5}{2}, \frac{5}{4}, 3\right) = \frac{5}{4} = \theta_2$

~~Replace~~ x_1 is leaving, Replace $x_1' = 4 - x_1$

پہلے کر لیں یا leave کروالیں کوئی فرق نہیں پڑتا $x_1 = 4 - x_1'$

Basic	x_1'	x_2''	x_3	x_4	x_5	Solution
z	0	-1	$5/2$	0	3	$109/2$
x_4	0	1	$1/2$	1	-1	$5/2$
x_1'	-1	-2	$3/2$	0	1	$-5/2$

x_2'' enters and x_1' leaves

Basic	x_1'	x_2''	x_3	x_4	x_5	sol
z	$1/2$	0	$13/4$	0	$5/2$	$223/4$
x_4	$-1/2$	0	$5/4$	1	$-1/2$	$15/4$
x_2''	$1/2$	1	$-3/4$	0	$-1/2$	$5/4$

$R_2 - R_3$ $\quad$ $R_1 + R_3$

Hence by back substitution

$x_1' = 0 \Rightarrow 4 - x_1 = 0 \Rightarrow x_1 = 4$

$x_2'' = 5/4 \Rightarrow 3 - x_2' = 5/4 \Rightarrow 3 - x_2 + 7 = \frac{5}{4} \Rightarrow x_2 = \frac{35}{4}$

Hence the optimal feasible solution is

$x_1 = 4, \quad x_2 = \frac{35}{4}, \quad x_3 = 0, \quad z = \frac{223}{4}$

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